

Problem 1.20 Find complex numbers $t = z_1 + z_2$ and $s = z_1 - z_2$, both in polar form, for each of the following pairs:

- (a) $z_1 = 2 + j3$ and $z_2 = 1 - j2$,
- (b) $z_1 = 3$ and $z_2 = -j3$,
- (c) $z_1 = 3\angle 30^\circ$ and $z_2 = 3\angle -30^\circ$,
- (d) $z_1 = 3\angle 30^\circ$ and $z_2 = 3\angle -150^\circ$.

Solution:

(a)

$$\begin{aligned}t &= z_1 + z_2 = (2 + j3) + (1 - j2) = 3 + j1, \\s &= z_1 - z_2 = (2 + j3) - (1 - j2) = 1 + j5 = 5.1 e^{j78.7^\circ}.\end{aligned}$$

(b)

$$\begin{aligned}t &= z_1 + z_2 = 3 - j3 = 4.24 e^{-j45^\circ}, \\s &= z_1 - z_2 = 3 + j3 = 4.24 e^{j45^\circ}.\end{aligned}$$

(c)

$$\begin{aligned}t &= z_1 + z_2 = 3\angle 30^\circ + 3\angle -30^\circ \\&= 3e^{j30^\circ} + 3e^{-j30^\circ} = (2.6 + j1.5) + (2.6 - j1.5) = 5.2, \\s &= z_1 - z_2 = 3e^{j30^\circ} - 3e^{-j30^\circ} = (2.6 + j1.5) - (2.6 - j1.5) = j3 = 3e^{j90^\circ}.\end{aligned}$$

(d)

$$\begin{aligned}t &= z_1 + z_2 = 3\angle 30^\circ + 3\angle -150^\circ = (2.6 + j1.5) + (-2.6 - j1.5) = 0, \\s &= z_1 - z_2 = (2.6 + j1.5) - (-2.6 - j1.5) = 5.2 + j3 = 6e^{j30^\circ}.\end{aligned}$$
