

Problem 1.29 The voltage source of the circuit shown in Fig. P1.29 is given by

$$v_s(t) = 25 \cos(4 \times 10^4 t - 45^\circ) \text{ (V)}.$$

Obtain an expression for $i_L(t)$, the current flowing through the inductor.

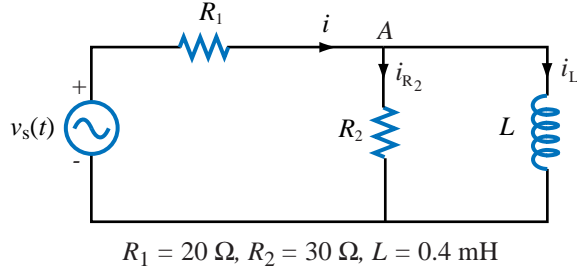


Figure P1.29:

Solution: Based on the given voltage expression, the phasor source voltage is

$$\tilde{V}_s = 25e^{-j45^\circ} \text{ (V)}. \quad (9)$$

The voltage equation for the left-hand side loop is

$$R_1 i + R_2 i_{R_2} = v_s \quad (10)$$

For the right-hand loop,

$$R_2 i_{R_2} = L \frac{di_L}{dt}, \quad (11)$$

and at node A,

$$i = i_{R_2} + i_L. \quad (12)$$

Next, we convert Eqs. (2)–(4) into phasor form:

$$R_1 \tilde{I} + R_2 \tilde{I}_{R_2} = \tilde{V}_s \quad (13)$$

$$R_2 \tilde{I}_{R_2} = j\omega L \tilde{I}_L \quad (14)$$

$$\tilde{I} = \tilde{I}_{R_2} + \tilde{I}_L \quad (15)$$

Upon combining (6) and (7) to solve for \tilde{I}_{R_2} in terms of \tilde{I} , we have:

$$\tilde{I}_{R_2} = \frac{j\omega L}{R_2 + j\omega L} \tilde{I}. \quad (16)$$

Substituting (8) in (5) and then solving for \tilde{I} leads to:

$$\begin{aligned}
 R_1 \tilde{I} + \frac{jR_2 \omega L}{R_2 + j\omega L} \tilde{I} &= \tilde{V}_s \\
 \tilde{I} \left(R_1 + \frac{jR_2 \omega L}{R_2 + j\omega L} \right) &= \tilde{V}_s \\
 \tilde{I} \left(\frac{R_1 R_2 + jR_1 \omega L + jR_2 \omega L}{R_2 + j\omega L} \right) &= \tilde{V}_s \\
 \tilde{I} &= \left(\frac{R_2 + j\omega L}{R_1 R_2 + j\omega L(R_1 + R_2)} \right) \tilde{V}_s.
 \end{aligned} \tag{17}$$

Combining (6) and (7) to solve for \tilde{I}_L in terms of \tilde{I} gives

$$\tilde{I}_L = \frac{R_2}{R_2 + j\omega L} \tilde{I}. \tag{18}$$

Combining (9) and (10) leads to

$$\begin{aligned}
 \tilde{I}_L &= \left(\frac{R_2}{R_2 + j\omega L} \right) \left(\frac{R_2 + j\omega L}{R_1 R_2 + j\omega L(R_1 + R_2)} \right) \tilde{V}_s \\
 &= \frac{R_2}{R_1 R_2 + j\omega L(R_1 + R_2)} \tilde{V}_s.
 \end{aligned}$$

Using (1) for \tilde{V}_s and replacing R_1, R_2, L and ω with their numerical values, we have

$$\begin{aligned}
 \tilde{I}_L &= \frac{30}{20 \times 30 + j4 \times 10^4 \times 0.4 \times 10^{-3}(20 + 30)} 25e^{-j45^\circ} \\
 &= \frac{30 \times 25}{600 + j800} e^{-j45^\circ} \\
 &= \frac{7.5}{6 + j8} e^{-j45^\circ} = \frac{7.5e^{-j45^\circ}}{10e^{j53.1^\circ}} = 0.75e^{-j98.1^\circ} \quad (\text{A}).
 \end{aligned}$$

Finally,

$$\begin{aligned}
 i_L(t) &= \Re[\tilde{I}_L e^{j\omega t}] \\
 &= 0.75 \cos(4 \times 10^4 t - 98.1^\circ) \quad (\text{A}).
 \end{aligned}$$