

Problem 1.4 A wave traveling along a string is given by

$$y(x, t) = 2 \sin(4\pi t + 10\pi x) \quad (\text{cm}),$$

where x is the distance along the string in meters and y is the vertical displacement. Determine: (a) the direction of wave travel, (b) the reference phase ϕ_0 , (c) the frequency, (d) the wavelength, and (e) the phase velocity.

Solution:

(a) We start by converting the given expression into a cosine function of the form given by (1.17):

$$y(x, t) = 2 \cos\left(4\pi t + 10\pi x - \frac{\pi}{2}\right) \quad (\text{cm}).$$

Since the coefficients of t and x both have the same sign, the wave is traveling in the negative x -direction.

(b) From the cosine expression, $\phi_0 = -\pi/2$.

(c) $\omega = 2\pi f = 4\pi$,

$$f = 4\pi/2\pi = 2 \text{ Hz}.$$

(d) $2\pi/\lambda = 10\pi$,

$$\lambda = 2\pi/10\pi = 0.2 \text{ m}.$$

(e) $u_p = f\lambda = 2 \times 0.2 = 0.4 \text{ (m/s)}$.
