

Problem 1.8 Two waves on a string are given by the following functions:

$$y_1(x, t) = 4 \cos(20t - 30x) \quad (\text{cm})$$

$$y_2(x, t) = -4 \cos(20t + 30x) \quad (\text{cm})$$

where x is in centimeters. The waves are said to interfere constructively when their superposition $|y_s| = |y_1 + y_2|$ is a maximum, and they interfere destructively when $|y_s|$ is a minimum.

- (a) What are the directions of propagation of waves $y_1(x, t)$ and $y_2(x, t)$?
- (b) At $t = (\pi/50)$ s, at what location x do the two waves interfere constructively, and what is the corresponding value of $|y_s|$?
- (c) At $t = (\pi/50)$ s, at what location x do the two waves interfere destructively, and what is the corresponding value of $|y_s|$?

Solution:

(a) $y_1(x, t)$ is traveling in positive x -direction. $y_2(x, t)$ is traveling in negative x -direction.

(b) At $t = (\pi/50)$ s, $y_s = y_1 + y_2 = 4[\cos(0.4\pi - 30x) - \cos(0.4\pi + 3x)]$. Using the formulas from Appendix C,

$$2 \sin x \sin y = \cos(x - y) - (\cos x + y),$$

we have

$$y_s = 8 \sin(0.4\pi) \sin 30x = 7.61 \sin 30x.$$

Hence,

$$|y_s|_{\max} = 7.61$$

and it occurs when $\sin 30x = 1$, or $30x = \frac{\pi}{2} + 2n\pi$, or $x = \left(\frac{\pi}{60} + \frac{2n\pi}{30}\right)$ cm, where $n = 0, 1, 2, \dots$.

- (c) $|y_s|_{\min} = 0$ and it occurs when $30x = n\pi$, or $x = \frac{n\pi}{30}$ cm.
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