

Problem 2.13 In addition to not dissipating power, a lossless line has two important features: (1) it is dispersionless (μ_p is independent of frequency) and (2) its characteristic impedance Z_0 is purely real. Sometimes, it is not possible to design a transmission line such that $R' \ll \omega L'$ and $G' \ll \omega C'$, but it is possible to choose the dimensions of the line and its material properties so as to satisfy the condition

$$R'C' = L'G' \quad (\text{distortionless line}).$$

Such a line is called a *distortionless* line because despite the fact that it is not lossless, it does nonetheless possess the previously mentioned features of the loss line. Show that for a distortionless line,

$$\alpha = R' \sqrt{\frac{C'}{L'}} = \sqrt{R'G'}, \quad \beta = \omega \sqrt{L'C'}, \quad Z_0 = \sqrt{\frac{L'}{C'}}.$$

Solution: Using the distortionless condition in Eq. (2.22) gives

$$\begin{aligned} \gamma &= \alpha + j\beta = \sqrt{(R' + j\omega L')(G' + j\omega C')} \\ &= \sqrt{L'C'} \sqrt{\left(\frac{R'}{L'} + j\omega\right) \left(\frac{G'}{C'} + j\omega\right)} \\ &= \sqrt{L'C'} \sqrt{\left(\frac{R'}{L'} + j\omega\right) \left(\frac{R'}{L'} + j\omega\right)} \\ &= \sqrt{L'C'} \left(\frac{R'}{L'} + j\omega\right) = R' \sqrt{\frac{C'}{L'}} + j\omega \sqrt{L'C'}. \end{aligned}$$

Hence,

$$\alpha = \Re(\gamma) = R' \sqrt{\frac{C'}{L'}}, \quad \beta = \Im(\gamma) = \omega \sqrt{L'C'}, \quad u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{L'C'}}.$$

Similarly, using the distortionless condition in Eq. (2.29) gives

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} = \sqrt{\frac{L'}{C'}} \sqrt{\frac{R'/L' + j\omega}{G'/C' + j\omega}} = \sqrt{\frac{L'}{C'}}.$$
