

Problem 2.2 A two-wire copper transmission line is embedded in a dielectric material with $\epsilon_r = 2.6$ and $\sigma = 2 \times 10^{-6}$ S/m. Its wires are separated by 3 cm and their radii are 1 mm each.

- (a) Calculate the line parameters R' , L' , G' , and C' at 2 GHz.
- (b) Compare your results with those based on CD Module 2.1. Include a printout of the screen display.

Solution:

(a) Given:

$$\begin{aligned}
 f &= 2 \times 10^9 \text{ Hz}, \\
 d &= 2 \times 10^{-3} \text{ m}, \\
 D &= 3 \times 10^{-2} \text{ m}, \\
 \sigma_c &= 5.8 \times 10^7 \text{ S/m (copper)}, \\
 \epsilon_r &= 2.6, \\
 \sigma &= 2 \times 10^{-6} \text{ S/m}, \\
 \mu &= \mu_c = \mu_0.
 \end{aligned}$$

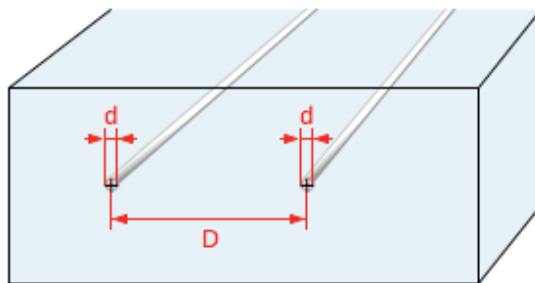
From Table 2-1:

$$\begin{aligned}
 R_s &= \sqrt{\pi f \mu_c / \sigma_c} \\
 &= [\pi \times 2 \times 10^9 \times 4\pi \times 10^{-7} / 5.8 \times 10^7]^{1/2} \\
 &= 1.17 \times 10^{-2} \Omega, \\
 R' &= \frac{2R_s}{\pi d} = \frac{2 \times 1.17 \times 10^{-2}}{2\pi \times 10^{-3}} = 3.71 \Omega/\text{m}, \\
 L' &= \frac{\mu}{\pi} \ln \left[(D/d) + \sqrt{(D/d)^2 - 1} \right] \\
 &= 1.36 \times 10^{-6} \text{ H/m}, \\
 G' &= \frac{\pi \sigma}{\ln[(D/d) + \sqrt{(D/d)^2 - 1}]} \\
 &= 1.85 \times 10^{-6} \text{ S/m}, \\
 C' &= \frac{G' \epsilon}{\sigma} \\
 &= \frac{1.85 \times 10^{-6} \times 8.85 \times 10^{-12} \times 2.6}{2 \times 10^{-6}} \\
 &= 2.13 \times 10^{-11} \text{ F/m}.
 \end{aligned}$$

(b) Solution via Module 2.1:

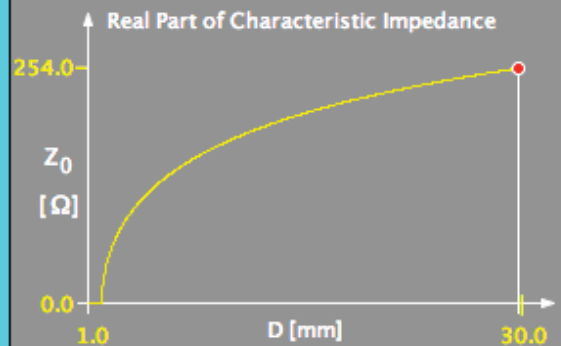
Module 2.1 Two-Wire Line

Select: Impedance vs. Distance D



Substrate
 $\epsilon_r = 2.6$
 $\sigma = 2.0E-6$ [S/m]

Wires
 $\sigma_c = 5.8E7$ [S/m]



Input

Wire Diameter $d = 2.0$ [mm]

Range

Centers distance $D = 30.0$ [mm]

Range

Frequency $f = 2.0E9$ [Hz]

Range

ϵ_r σ [S/m] σ_c [S/m]
 2.6 2E-6 5.8E7

Update

Output

$f = 2.0$ [GHz]

Structure Data

$d = 2.0$ [mm] $D / d = 15.0$
 $D = 30.0$ [mm]

$Z_0 = 253.037142 - j 0.026617$ [Ω]

$C' = 21.241303$ [pF/m]

$L' = 1.360034$ [μH/m]

$R' = 3.713907$ [Ω/m]

$G' = 2.0E-6$ [S/m]

$\lambda_0 = 0.15$ [m] in vacuum

$\lambda = 9.3026$ [cm] in guide

$\alpha = 0.007572$ [Np/m]

$\beta = 67.542213$ [rad/m]