

**Problem 2.23** A load with impedance  $Z_L = (25 - j50) \Omega$  is to be connected to a lossless transmission line with characteristic impedance  $Z_0$ , with  $Z_0$  chosen such that the standing-wave ratio is the smallest possible. What should  $Z_0$  be?

**Solution:** Since  $S$  is monotonic with  $|\Gamma|$  (i.e., a plot of  $S$  vs.  $|\Gamma|$  is always increasing), the value of  $Z_0$  which gives the minimum possible  $S$  also gives the minimum possible  $|\Gamma|$ , and, for that matter, the minimum possible  $|\Gamma|^2$ . A necessary condition for a minimum is that its derivative be equal to zero:

$$\begin{aligned} 0 = \frac{\partial}{\partial Z_0} |\Gamma|^2 &= \frac{\partial}{\partial Z_0} \frac{|R_L + jX_L - Z_0|^2}{|R_L + jX_L + Z_0|^2} \\ &= \frac{\partial}{\partial Z_0} \frac{(R_L - Z_0)^2 + X_L^2}{(R_L + Z_0)^2 + X_L^2} = \frac{4R_L(Z_0^2 - (R_L^2 + X_L^2))}{((R_L + Z_0)^2 + X_L^2)^2}. \end{aligned}$$

Therefore,  $Z_0^2 = R_L^2 + X_L^2$  or

$$Z_0 = |Z_L| = \sqrt{(25^2 + (-50)^2)} = 55.9 \Omega.$$

A mathematically precise solution will also demonstrate that this point is a minimum (by calculating the second derivative, for example). Since the endpoints of the range may be local minima or maxima without the derivative being zero there, the endpoints (namely  $Z_0 = 0 \Omega$  and  $Z_0 = \infty \Omega$ ) should be checked also.

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