Problem 2.3 Show that the transmission line model shown in Fig. P2.3 yields the same telegrapher's equations given by Eqs. (2.14) and (2.16).

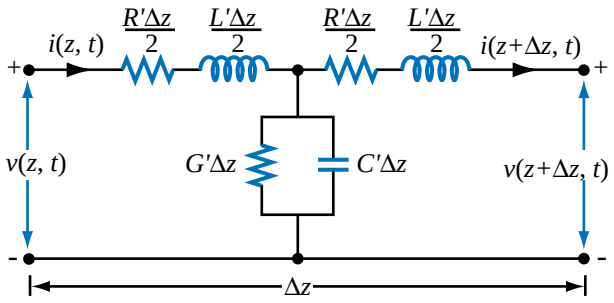

Figure P2.3: Transmission line model.

Solution: The voltage at the central upper node is the same whether it is calculated from the left port or the right port:

$$\begin{aligned} v(z+\tfrac{1}{2}\Delta z,t) &= v(z,t) - \tfrac{1}{2}R'\Delta z\, i(z,t) - \tfrac{1}{2}L'\Delta z\frac{\partial}{\partial t}i(z,t) \\ &= v(z+\Delta z,t) + \tfrac{1}{2}R'\Delta z\, i(z+\Delta z,t) + \tfrac{1}{2}L'\Delta z\frac{\partial}{\partial t}i(z+\Delta z,t). \end{aligned}$$

Recognizing that the current through the $G' \parallel C'$ branch is $i(z,t) - i(z+\Delta z,t)$ (from Kirchhoff's current law), we can conclude that

$$i(z,t) - i(z+\Delta z,t) = G'\Delta z\, v(z+\tfrac{1}{2}\Delta z,t) + C'\Delta z\frac{\partial}{\partial t}v(z+\tfrac{1}{2}\Delta z,t).$$

From both of these equations, the proof is completed by following the steps outlined in the text, ie. rearranging terms, dividing by Δz, and taking the limit as $\Delta z \to 0$.