

Problem 2.32 A 6-m section of $150\text{-}\Omega$ lossless line is driven by a source with

$$v_g(t) = 5 \cos(8\pi \times 10^7 t - 30^\circ) \quad (\text{V})$$

and $Z_g = 150\text{ }\Omega$. If the line, which has a relative permittivity $\epsilon_r = 2.25$, is terminated in a load $Z_L = (150 - j50)\text{ }\Omega$, determine:

- (a) λ on the line.
- (b) The reflection coefficient at the load.
- (c) The input impedance.
- (d) The input voltage \tilde{V}_i .
- (e) The time-domain input voltage $v_i(t)$.
- (f) Quantities in (a) to (d) using CD Modules 2.4 or 2.5.

Solution:

$$v_g(t) = 5 \cos(8\pi \times 10^7 t - 30^\circ) \text{ V},$$

$$\tilde{V}_g = 5e^{-j30^\circ} \text{ V}.$$

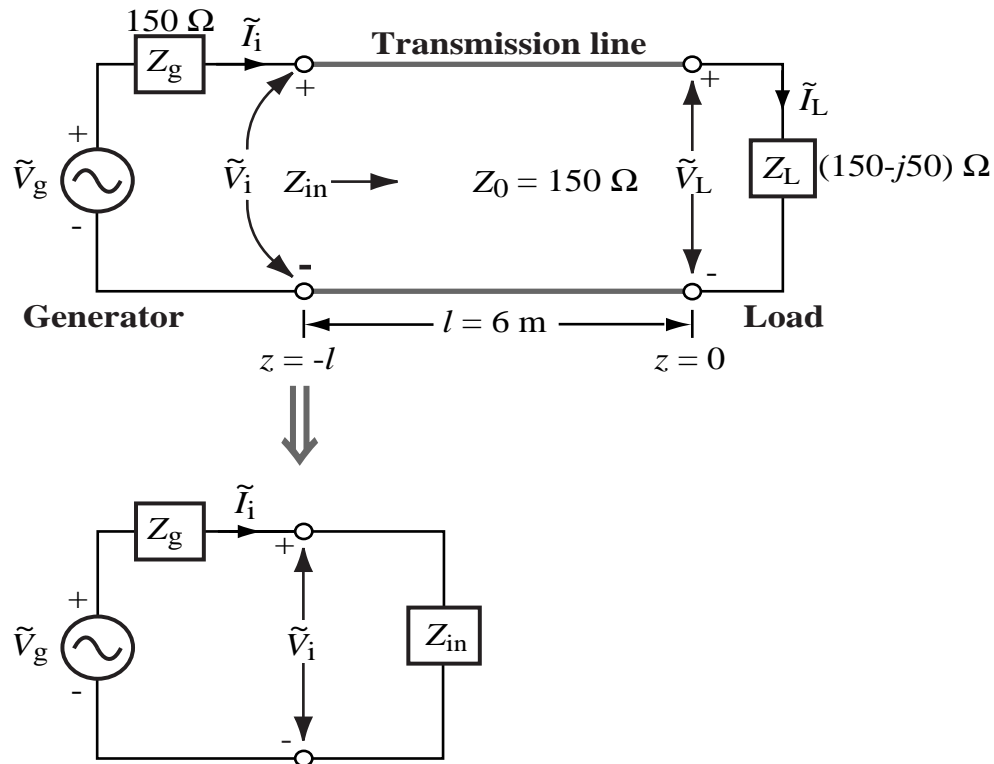


Figure P2.32: Circuit for Problem 2.32.

(a)

$$\begin{aligned}u_p &= \frac{c}{\sqrt{\epsilon_r}} = \frac{3 \times 10^8}{\sqrt{2.25}} = 2 \times 10^8 \text{ (m/s)}, \\ \lambda &= \frac{u_p}{f} = \frac{2\pi u_p}{\omega} = \frac{2\pi \times 2 \times 10^8}{8\pi \times 10^7} = 5 \text{ m}, \\ \beta &= \frac{\omega}{u_p} = \frac{8\pi \times 10^7}{2 \times 10^8} = 0.4\pi \text{ (rad/m)}, \\ \beta l &= 0.4\pi \times 6 = 2.4\pi \text{ (rad)}.\end{aligned}$$

Since this exceeds 2π (rad), we can subtract 2π , which leaves a remainder $\beta l = 0.4\pi$ (rad).

(b) $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{150 - j50 - 150}{150 - j50 + 150} = \frac{-j50}{300 - j50} = 0.16e^{-j80.54^\circ}.$

(c)

$$\begin{aligned}Z_{in} &= Z_0 \left[\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right] \\ &= 150 \left[\frac{(150 - j50) + j150 \tan(0.4\pi)}{150 + j(150 - j50) \tan(0.4\pi)} \right] = (115.70 + j27.42) \Omega.\end{aligned}$$

(d)

$$\begin{aligned}\tilde{V}_i &= \frac{\tilde{V}_g Z_{in}}{Z_g + Z_{in}} = \frac{5e^{-j30^\circ} (115.7 + j27.42)}{150 + 115.7 + j27.42} \\ &= 5e^{-j30^\circ} \left(\frac{115.7 + j27.42}{265.7 + j27.42} \right) \\ &= 5e^{-j30^\circ} \times 0.44e^{j7.44^\circ} = 2.2e^{-j22.56^\circ} \text{ (V)}.\end{aligned}$$

(e)

$$v_i(t) = \Re[\tilde{V}_i e^{j\omega t}] = \Re[2.2e^{-j22.56^\circ} e^{j\omega t}] = 2.2 \cos(8\pi \times 10^7 t - 22.56^\circ) \text{ V}.$$

Module 2.4
Transmission Line Simulator
Options: Set Input / Output

d =

$Z_g = 150.0 + j 0.0 \ \Omega$
 $V_g = 4.33 - j 2.5 \text{ V}$

$Z_0 = 150.0 + j 0.0 \ \Omega$
 $\epsilon_r = 2.25$

$f = 40.0 \text{ MHz}$
 $\lambda = 5.0 \text{ m}$

$Z_L = 150.0 - j 50.0 \ \Omega$

d = 1.2 λ = 6.0 m

d = 0

Set Line
Length units: ☐ [λ] ☒ [m]

Low Loss Approximation
Characteristic Impedance $Z_0 = 150 \ \Omega$
Frequency $f = 4E7 \text{ Hz}$
Relative Permittivity $\epsilon_r = 2.25$
Line Length $l = 6 \text{ [m]}$
Update

$Z_L = 150 + j -50 \ \Omega$
☒ Impedance ☐ Admittance
Update

Set Generator
 $V_g = 4.33 + j -2.5 \text{ V}$
 $Z_g = 150 + j 0.0 \ \Omega$
Update

Output
Transmission Line Data 1
Cursor d = 1.2 λ = 6.0 m

Impedance $Z(d) = 115.702409 + j 27.423507 \ \Omega$
= 118.907931 \angle 0.2327 rad

Admittance $Y(d) = 0.008183 - j 0.00194 \text{ S}$
= 0.00841 \angle -0.2327 rad

Reflection Coefficient $\Gamma_d = -0.11718185 + j 0.11530585$
= 0.16439899 \angle 2.364264 rad
= 0.16439899 \angle 135.462322 $^\circ$

Voltage $V(d) = 2.055434 - j 0.853886 \text{ V}$
= 2.225742 \angle -0.3937 rad

Current $I(d) = 0.015164 - j 0.010974 \text{ A}$
= 0.018718 \angle -0.6265 rad

Power Flow $P_{av} = 20.269378 \text{ mW}$

5 cos (-30)

5 sin(-30)