Problem 2.33 Two half-wave dipole antennas, each with an impedance of 75 Ω, are connected in parallel through a pair of transmission lines, and the combination is connected to a feed transmission line, as shown in Fig. P2.33.

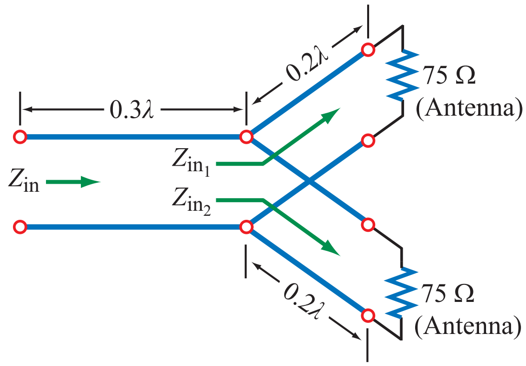

Figure P2.33: Circuit for Problem 2.33.

All lines are 50 Ω and lossless.

(a) Calculate Z_{in_1}, the input impedance of the antenna-terminated line, at the parallel juncture.

(b) Combine Z_{in_1} and Z_{in_2} in parallel to obtain Z'_L, the effective load impedance of the feedline.

(c) Calculate Z_{in} of the feedline.

Solution:

(a)

$$Z_{in_1} = Z_0 \left[\frac{Z_{L_1} + jZ_0 \tan\beta l_1}{Z_0 + jZ_{L_1} \tan\beta l_1} \right]$$

$$= 50 \left\{ \frac{75 + j50 \tan[(2\pi/\lambda)(0.2\lambda)]}{50 + j75 \tan[(2\pi/\lambda)(0.2\lambda)]} \right\} = (35.20 - j8.62)\ \Omega.$$

(b)

$$Z'_L = \frac{Z_{in_1} Z_{in_2}}{Z_{in_1} + Z_{in_2}} = \frac{(35.20 - j8.62)^2}{2(35.20 - j8.62)} = (17.60 - j4.31)\ \Omega.$$

(c)

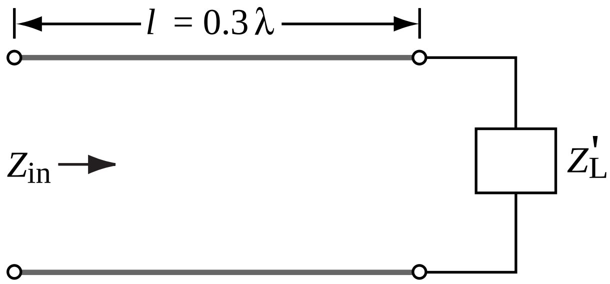

Figure P2.33: (b) Equivalent circuit.

$$Z_{in} = 50 \left\{ \frac{(17.60 - j4.31) + j50\tan[(2\pi/\lambda)(0.3\lambda)]}{50 + j(17.60 - j4.31)\tan[(2\pi/\lambda)(0.3\lambda)]} \right\} = (107.57 - j56.7)\ \Omega.$$