Problem 2.34 A 50-Ω lossless line is terminated in a load impedance $Z_L = (30 - j20)$ Ω.

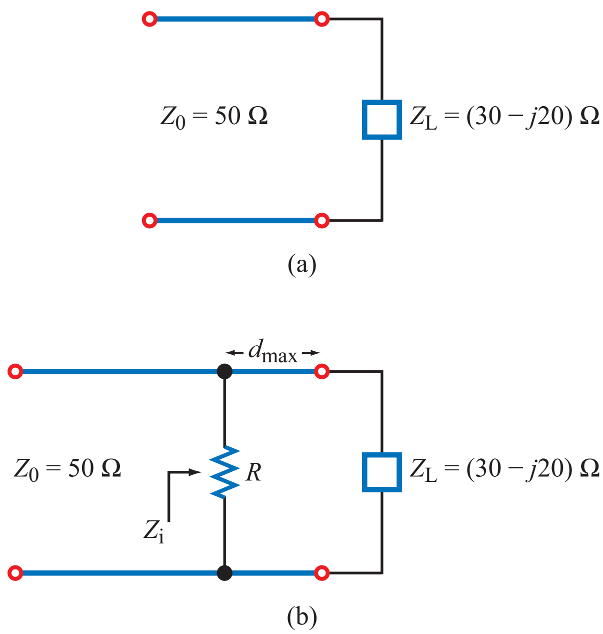

Figure P2.34: Circuit for Problem 2.34.

(a) Calculate Γ and S.

(b) It has been proposed that by placing an appropriately selected resistor across the line at a distance d_{max} from the load (as shown in Fig. P2.34(b)), where d_{max} is the distance from the load of a voltage maximum, then it is possible to render $Z_i = Z_0$, thereby eliminating reflection back to the end. Show that the proposed approach is valid and find the value of the shunt resistance.

Solution:

(a)

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{30 - j20 - 50}{30 - j20 + 50} = \frac{-20 - j20}{80 - j20} = \frac{-(20 + j20)}{80 - j20} = 0.34e^{-j121^\circ}.$$

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.34}{1 - 0.34} = 2.$$

(b) We start by finding d_{max}, the distance of the voltage maximum nearest to the load. Using (2.70) with $n = 1$,

$$d_{max} = \frac{\theta_r \lambda}{4\pi} + \frac{\lambda}{2} = \left(\frac{-121^\circ \pi}{180^\circ}\right)\frac{\lambda}{4\pi} + \frac{\lambda}{2} = 0.33\lambda.$$

Applying (2.79) at $d = d_{\max} = 0.33\lambda$, for which $\beta l = (2\pi/\lambda) \times 0.33\lambda = 2.07$ radians, the value of Z_{in} before adding the shunt resistance is:

$$\begin{aligned} Z_{\text{in}} &= Z_0 \left(\frac{Z_{\text{L}} + jZ_0 \tan \beta l}{Z_0 + jZ_{\text{L}} \tan \beta l} \right) \\ &= 50 \left(\frac{(30 - j20) + j50 \tan 2.07}{50 + j(30 - j20) \tan 2.07} \right) = (102 + j0)\ \Omega. \end{aligned}$$

Thus, at the location A (at a distance $d_{\max}$ from the load), the input impedance is purely real. If we add a shunt resistor R in parallel such that the combination is equal to Z_0, then the new Z_{in} at any point to the left of that location will be equal to Z_0.

Hence, we need to select R such that

$$\frac{1}{R} + \frac{1}{102} = \frac{1}{50}$$

or $R = 98\ \Omega$.