Problem 2.40 A 100-MHz FM broadcast station uses a 300-Ω transmission line between the transmitter and a tower-mounted half-wave dipole antenna. The antenna impedance is 73 Ω. You are asked to design a quarter-wave transformer to match the antenna to the line.

(a) Determine the electrical length and characteristic impedance of the quarter-wave section.

(b) If the quarter-wave section is a two-wire line with $D = 2.5$ cm, and the wires are embedded in polystyrene with $\varepsilon_r = 2.6$, determine the physical length of the quarter-wave section and the radius of the two wire conductors.

Solution:

(a) For a match condition, the input impedance of a load must match that of the transmission line attached to the generator. A line of electrical length $\lambda/4$ can be used. From Eq. (2.97), the impedance of such a line should be

$$Z_0 = \sqrt{Z_{in} Z_L} = \sqrt{300 \times 73} = 148\ \Omega.$$

(b)

$$\frac{\lambda}{4} = \frac{u_p}{4f} = \frac{c}{4\sqrt{\varepsilon_r} f} = \frac{3 \times 10^8}{4\sqrt{2.6} \times 100 \times 10^6} = 0.465\ \text{m},$$

and, from Table 2-2,

$$Z_0 = \frac{120}{\sqrt{\varepsilon}} \ln\left[\left(\frac{D}{d}\right) + \sqrt{\left(\frac{D}{d}\right)^2 - 1}\right]\ \Omega.$$

Hence,

$$\ln\left[\left(\frac{D}{d}\right) + \sqrt{\left(\frac{D}{d}\right)^2 - 1}\right] = \frac{148\sqrt{2.6}}{120} = 1.99,$$

which leads to

$$\left(\frac{D}{d}\right) + \sqrt{\left(\frac{D}{d}\right)^2 - 1} = 7.31,$$

and whose solution is $D/d = 3.73$. Hence, $d = D/3.73 = 2.5\ \text{cm}/3.73 = 0.67$ cm.