

Problem 2.41 A $50\text{-}\Omega$ lossless line of length $l = 0.375\lambda$ connects a 300-MHz generator with $\tilde{V}_g = 300\text{ V}$ and $Z_g = 50\text{ }\Omega$ to a load Z_L . Determine the time-domain current through the load for:

- (a) $Z_L = (50 - j50)\text{ }\Omega$
- (b) $Z_L = 50\text{ }\Omega$
- (c) $Z_L = 0$ (short circuit)

For (a), verify your results by deducing the information you need from the output products generated by CD Module 2.4.

Solution:

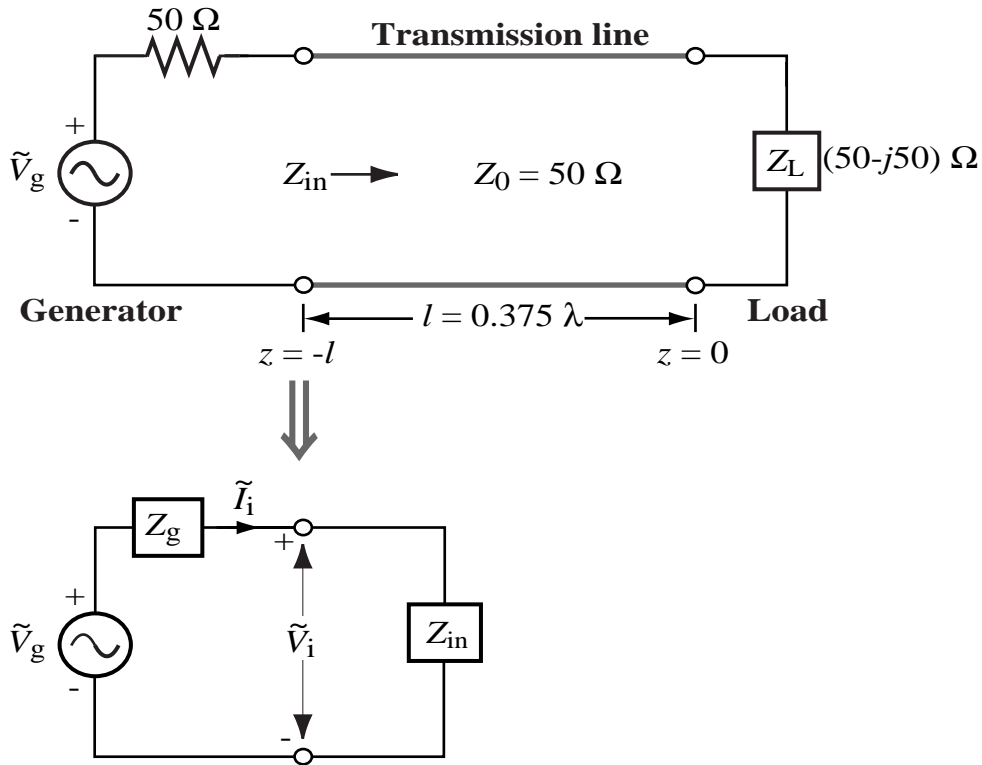


Figure P2.41: Circuit for Problem 2.41(a).

- (a) $Z_L = (50 - j50)\text{ }\Omega$, $\beta l = \frac{2\pi}{\lambda} \times 0.375\lambda = 2.36\text{ (rad)} = 135^\circ$.

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 - j50 - 50}{50 - j50 + 50} = \frac{-j50}{100 - j50} = 0.45 e^{-j63.43^\circ}.$$

Application of Eq. (2.79) gives:

$$Z_{\text{in}} = Z_0 \left[\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right] = 50 \left[\frac{(50 - j50) + j50 \tan 135^\circ}{50 + j(50 - j50) \tan 135^\circ} \right] = (100 + j50) \Omega.$$

Using Eq. (2.82) gives

$$\begin{aligned} V_0^+ &= \left(\frac{\tilde{V}_g Z_{\text{in}}}{Z_g + Z_{\text{in}}} \right) \left(\frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}} \right) \\ &= \frac{300(100 + j50)}{50 + (100 + j50)} \left(\frac{1}{e^{j135^\circ} + 0.45 e^{-j63.43^\circ} e^{-j135^\circ}} \right) \\ &= 150 e^{-j135^\circ} \quad (\text{V}), \\ \tilde{I}_L &= \frac{V_0^+}{Z_0} (1 - \Gamma) = \frac{150 e^{-j135^\circ}}{50} (1 - 0.45 e^{-j63.43^\circ}) = 2.68 e^{-j108.44^\circ} \quad (\text{A}), \\ i_L(t) &= \Re \{ \tilde{I}_L e^{j\omega t} \} \\ &= \Re \{ 2.68 e^{-j108.44^\circ} e^{j6\pi \times 10^8 t} \} \\ &= 2.68 \cos(6\pi \times 10^8 t - 108.44^\circ) \quad (\text{A}). \end{aligned}$$

(b)

$$\begin{aligned} Z_L &= 50 \Omega, \\ \Gamma &= 0, \\ Z_{\text{in}} &= Z_0 = 50 \Omega, \\ V_0^+ &= \frac{300 \times 50}{50 + 50} \left(\frac{1}{e^{j135^\circ} + 0} \right) = 150 e^{-j135^\circ} \quad (\text{V}), \\ \tilde{I}_L &= \frac{V_0^+}{Z_0} = \frac{150}{50} e^{-j135^\circ} = 3 e^{-j135^\circ} \quad (\text{A}), \\ i_L(t) &= \Re \{ 3 e^{-j135^\circ} e^{j6\pi \times 10^8 t} \} = 3 \cos(6\pi \times 10^8 t - 135^\circ) \quad (\text{A}). \end{aligned}$$

(c)

$$\begin{aligned} Z_L &= 0, \\ \Gamma &= -1, \\ Z_{\text{in}} &= Z_0 \left(\frac{0 + jZ_0 \tan 135^\circ}{Z_0 + 0} \right) = jZ_0 \tan 135^\circ = -j50 \quad (\Omega), \\ V_0^+ &= \frac{300(-j50)}{50 - j50} \left(\frac{1}{e^{j135^\circ} - e^{-j135^\circ}} \right) = 150 e^{-j135^\circ} \quad (\text{V}), \end{aligned}$$

$$\tilde{I}_L = \frac{V_0^+}{Z_0} [1 - \Gamma] = \frac{150 e^{-j135^\circ}}{50} [1 + 1] = 6 e^{-j135^\circ} \quad (\text{A}),$$

$$i_L(t) = 6 \cos(6\pi \times 10^8 t - 135^\circ) \quad (\text{A}).$$

From output of Module 2.4, at $d = 0$ (load)

$$\tilde{I}(d) = 2.68 \angle -1.89 \text{ rad},$$

which corresponds to

$$\tilde{I}(d) = 2.68 \angle -108.29^\circ.$$

The equivalent time-domain current at $f = 300$ MHz is

$$i_L(t) = 2.68 \cos(6\pi \times 10^8 t - 108.29^\circ) \quad (\text{A}).$$

