Problem 2.42 A generator with $\widetilde{V}_g = 300$ V and $Z_g = 50\ \Omega$ is connected to a load $Z_L = 75\ \Omega$ through a 50-Ω lossless line of length $l = 0.15\lambda$.

(a) Compute Z_{in}, the input impedance of the line at the generator end.

(b) Compute $\widetilde{I}_i$ and $\widetilde{V}_i$.

(c) Compute the time-average power delivered to the line, $P_{in} = \frac{1}{2}\mathfrak{Re}[\widetilde{V}_i\widetilde{I}_i^*]$.

(d) Compute $\widetilde{V}_L$, $\widetilde{I}_L$, and the time-average power delivered to the load, $P_L = \frac{1}{2}\mathfrak{Re}[\widetilde{V}_L\widetilde{I}_L^*]$. How does P_{in} compare to P_L? Explain.

(e) Compute the time-average power delivered by the generator, P_g, and the time-average power dissipated in Z_g. Is conservation of power satisfied?

Solution:

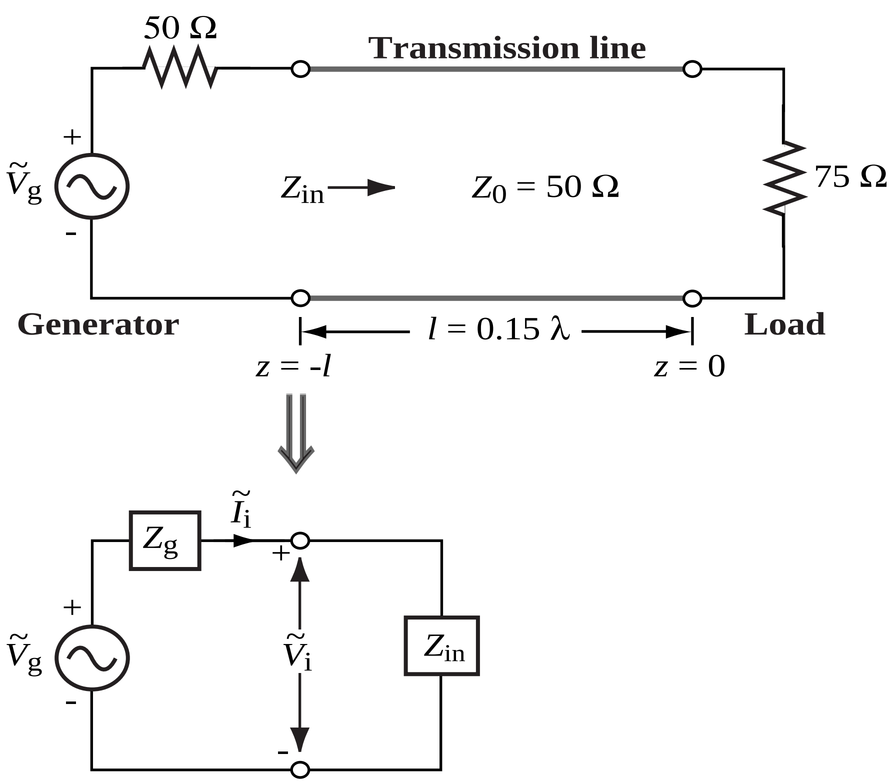

Figure P2.42: Circuit for Problem 2.42.

(a)

$$\beta l = \frac{2\pi}{\lambda} \times 0.15\lambda = 54°,$$

$$Z_{\text{in}} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right] = 50 \left[\frac{75 + j50\tan 54^\circ}{50 + j75\tan 54^\circ}\right] = (41.25 - j16.35)\ \Omega.$$

(b)

$$\widetilde{I}_i = \frac{\widetilde{V}_g}{Z_g + Z_{\text{in}}} = \frac{300}{50 + (41.25 - j16.35)} = 3.24\,e^{j10.16^\circ} \quad \text{(A)},$$

$$\widetilde{V}_i = \widetilde{I}_i Z_{\text{in}} = 3.24\,e^{j10.16^\circ}(41.25 - j16.35) = 143.6\,e^{-j11.46^\circ} \quad \text{(V)}.$$

(c)

$$P_{\text{in}} = \frac{1}{2}\mathfrak{Re}[\widetilde{V}_i \widetilde{I}_i^*] = \frac{1}{2}\mathfrak{Re}[143.6\,e^{-j11.46^\circ} \times 3.24\,e^{-j10.16^\circ}]$$
$$= \frac{143.6 \times 3.24}{2}\cos(21.62^\circ) = 216 \quad \text{(W)}.$$

(d)

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{75 - 50}{75 + 50} = 0.2,$$

$$V_0^+ = \widetilde{V}_i \left(\frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}}\right) = \frac{143.6\,e^{-j11.46^\circ}}{e^{j54^\circ} + 0.2\,e^{-j54^\circ}} = 150e^{-j54^\circ} \quad \text{(V)},$$

$$\widetilde{V}_L = V_0^+(1 + \Gamma) = 150e^{-j54^\circ}(1 + 0.2) = 180e^{-j54^\circ} \quad \text{(V)},$$

$$\widetilde{I}_L = \frac{V_0^+}{Z_0}(1 - \Gamma) = \frac{150e^{-j54^\circ}}{50}(1 - 0.2) = 2.4\,e^{-j54^\circ} \quad \text{(A)},$$

$$P_L = \frac{1}{2}\mathfrak{Re}[\widetilde{V}_L \widetilde{I}_L^*] = \frac{1}{2}\mathfrak{Re}[180e^{-j54^\circ} \times 2.4\,e^{j54^\circ}] = 216 \quad \text{(W)}.$$

$P_L = P_{\text{in}}$, which is as expected because the line is lossless; power input to the line ends up in the load.

(e)

Power delivered by generator:

$$P_g = \frac{1}{2}\mathfrak{Re}[\widetilde{V}_g \widetilde{I}_i] = \frac{1}{2}\mathfrak{Re}[300 \times 3.24\,e^{j10.16^\circ}] = 486\cos(10.16^\circ) = 478.4 \quad \text{(W)}.$$

Power dissipated in Z_g:

$$P_{Z_g} = \frac{1}{2}\mathfrak{Re}[\widetilde{I}_i \widetilde{V}_{Z_g}] = \frac{1}{2}\mathfrak{Re}[\widetilde{I}_i \widetilde{I}_i^* Z_g] = \frac{1}{2}|\widetilde{I}_i|^2 Z_g = \frac{1}{2}(3.24)^2 \times 50 = 262.4 \quad \text{(W)}.$$

Note 1: $P_g = P_{Z_g} + P_{\text{in}} = 478.4$ W.