Problem 2.43 If the two-antenna configuration shown in Fig. P2.43 is connected to a generator with $\widetilde{V}_g = 250$ V and $Z_g = 50\ \Omega$, how much average power is delivered to each antenna?

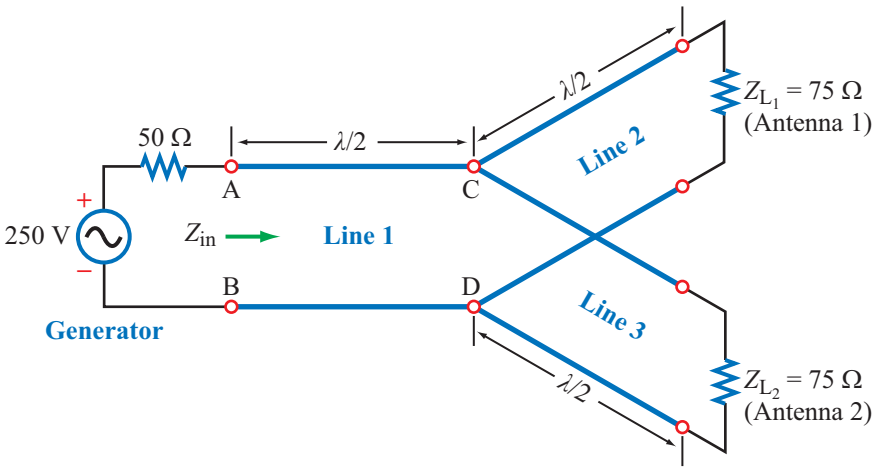

Figure P2.43: Antenna configuration for Problem 2.43.

Solution: Since line 2 is $\lambda/2$ in length, the input impedance is the same as $Z_{L_1} = 75\ \Omega$. The same is true for line 3. At junction C–D, we now have two 75-Ω impedances in parallel, whose combination is $75/2 = 37.5\ \Omega$. Line 1 is $\lambda/2$ long. Hence at A–C, input impedance of line 1 is 37.5 Ω, and

$$\widetilde{I}_i = \frac{\widetilde{V}_g}{Z_g + Z_{in}} = \frac{250}{50 + 37.5} = 2.86 \quad \text{(A)},$$

$$P_{in} = \frac{1}{2}\mathfrak{Re}[\widetilde{I}_i \widetilde{V}_i^*] = \frac{1}{2}\mathfrak{Re}[\widetilde{I}_i \widetilde{I}_i^* \widetilde{Z}_{in}^*] = \frac{(2.86)^2 \times 37.5}{2} = 153.37 \quad \text{(W)}.$$

This is divided equally between the two antennas. Hence, each antenna receives $\frac{153.37}{2} = 76.68$ (W).