

Problem 2.45 The circuit shown in Fig. P2.45 consists of a $100\text{-}\Omega$ lossless transmission line terminated in a load with $Z_L = (50 + j100)\text{ }\Omega$. If the peak value of the load voltage was measured to be $|\tilde{V}_L| = 12\text{ V}$, determine:

- (a) the time-average power dissipated in the load,
- (b) the time-average power incident on the line,
- (c) the time-average power reflected by the load.

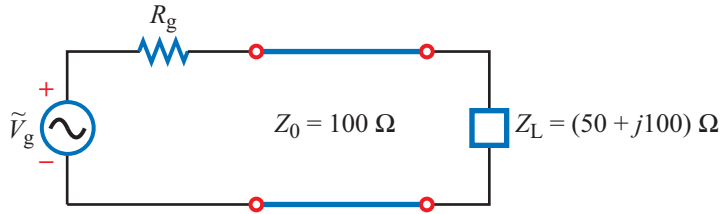


Figure P2.45: Circuit for Problem 2.45.

Solution:

(a)

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 + j100 - 100}{50 + j100 + 100} = \frac{-50 + j100}{150 + j100} = 0.62e^{j82.9^\circ}.$$

The time average power dissipated in the load is:

$$\begin{aligned} P_{av} &= \frac{1}{2} |\tilde{I}_L|^2 R_L \\ &= \frac{1}{2} \left| \frac{\tilde{V}_L}{Z_L} \right|^2 R_L \\ &= \frac{1}{2} \frac{|\tilde{V}_L|^2}{|Z_L|^2} R_L = \frac{1}{2} \times 12^2 \times \frac{50}{50^2 + 100^2} = 0.29\text{ W}. \end{aligned}$$

(b)

$$P_{av} = P_{av}^i (1 - |\Gamma|^2)$$

Hence,

$$P_{av}^i = \frac{P_{av}}{1 - |\Gamma|^2} = \frac{0.29}{1 - 0.62^2} = 0.47\text{ W}.$$

(c)

$$P_{av}^r = -|\Gamma|^2 P_{av}^i = -(0.62)^2 \times 0.47 = -0.18\text{ W}.$$
