Problem 2.46 An antenna with a load impedance

$$Z_L = (75 + j25)\ \Omega$$

is connected to a transmitter through a 50-Ω lossless transmission line. If under matched conditions (50-Ω load) the transmitter can deliver 20 W to the load, how much power can it deliver to the antenna? Assume that $Z_g = Z_0$.

Solution: From Eqs. (2.82) and (2.76),

$$\begin{aligned}
V_0^+ &= \left(\frac{\widetilde{V}_g Z_{in}}{Z_g + Z_{in}}\right)\left(\frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}}\right) \\
&= \left\{\frac{\widetilde{V}_g Z_0\left[(1+\Gamma e^{-j2\beta l})/(1-\Gamma e^{-j2\beta l})\right]}{Z_0 + Z_0\left[(1+\Gamma e^{-j2\beta l})/(1-\Gamma e^{-j2\beta l})\right]}\right\}\left(\frac{e^{-j\beta l}}{1+\Gamma e^{-j2\beta l}}\right) \\
&= \frac{\widetilde{V}_g e^{-j\beta l}}{(1-\Gamma e^{-j2\beta l}) + (1+\Gamma e^{-j2\beta l})} \\
&= \frac{\widetilde{V}_g e^{-j\beta l}}{(1-\Gamma e^{-j2\beta l}) + (1+\Gamma e^{-j2\beta l})} = \tfrac{1}{2}\widetilde{V}_g e^{-j\beta l}.
\end{aligned}$$

Thus, in Eq. (2.106),

$$P_{av} = \frac{|V_0^+|^2}{2Z_0}(1-|\Gamma|^2) = \frac{|\frac{1}{2}\widetilde{V}_g e^{-j\beta l}|^2}{2Z_0}(1-|\Gamma|^2) = \frac{|\widetilde{V}_g|^2}{8Z_0}(1-|\Gamma|^2).$$

Under the matched condition, $|\Gamma| = 0$ and $P_L = 20$ W, so $|\widetilde{V}_g|^2/8Z_0 = 20$ W.
When $Z_L = (75 + j25)\ \Omega$, from Eq. (2.59a),

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(75+j25)\ \Omega - 50\ \Omega}{(75+j25)\ \Omega + 50\ \Omega} = 0.277e^{j33.6^\circ},$$

so $P_{av} = 20\text{ W}\ (1-|\Gamma|^2) = 20\text{ W}\ (1-0.277^2) = 18.46$ W.