Problem 2.77 In response to a step voltage, the voltage waveform shown in Fig. P2.77 was observed at the sending end of a lossless transmission line with $R_g = 50\ \Omega$, $Z_0 = 50\ \Omega$, and $\varepsilon_r = 2.25$. Determine the following:

(a) The generator voltage.

(b) The length of the line.

(c) The load impedance.

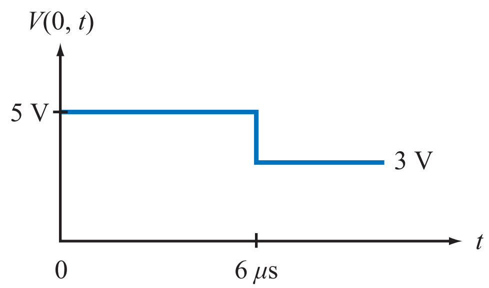

Figure P2.77: Voltage waveform for Problems 2.77 and 2.79.

Solution:

(a) From the figure, $V_1^+ = 5$ V. Applying Eq. (2.149b),

$$V_1^+ = \frac{V_g Z_0}{R_g + Z_0} = \frac{V_g Z_0}{Z_0 + Z_0} = \frac{V_g}{2},$$

which gives $V_g = 2V_1^+ = 10$ V.

(b) $u_p = \dfrac{c}{\sqrt{\varepsilon_r}} = \dfrac{3 \times 10^8}{\sqrt{2.25}} = 2 \times 10^8$ m/s. The first change in the waveform occurs at $\Delta t = 6\ \mu$s. But $\Delta t = 2l/u_p$. Hence,

$$l = \frac{\Delta t \mu_p}{2} = \frac{6 \times 10^{-6}}{2} \times 2 \times 10^8 = 600 \text{ m}.$$

(c) Since $R_g = Z_0$, $\Gamma_g = 0$. Hence $V_2^+ = 0$ and the change in level from 5 V down to 3 V is due to $V_1^- = -2$ V. But

$$V_1^- = \Gamma_L V_1^+, \qquad \text{or} \quad \Gamma_L = \frac{V_1^-}{V_1^+} = \frac{-2}{5} = -0.4.$$

From

$$Z_L = Z_0 \left(\frac{1 + \Gamma_L}{1 - \Gamma_L} \right) = 50 \left(\frac{1 - 0.4}{1 + 0.4} \right) = 21.43\ \Omega.$$