Problem 2.82 In response to a step voltage, the voltage waveform shown in Fig. P2.82 was observed at the midpoint of a lossless transmission line with $Z_0 = 50\ \Omega$ and $u_p = 2 \times 10^8$ m/s. Determine: (a) the length of the line, (b) Z_L, (c) R_g, and (d) V_g.

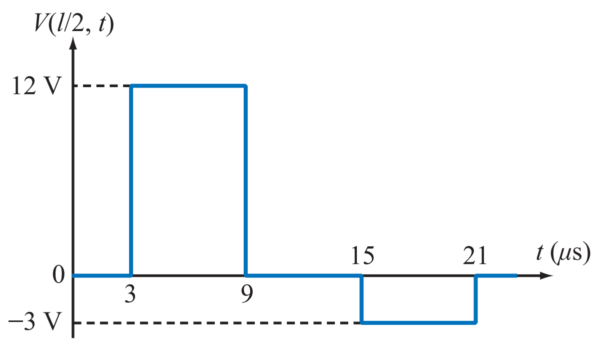

Figure P2.82: Circuit for Problem 2.82.

Solution:

(a) Since it takes 3 μs to reach the middle of the line, the line length must be

$$l = 2(3 \times 10^{-6} \times u_p) = 2 \times 3 \times 10^{-6} \times 2 \times 10^8 = 1200 \text{ m}.$$

(b) From the voltage waveform shown in the figure, the duration of the first rectangle is 6 μs, representing the time it takes the incident voltage V_1^+ to travel from the midpoint of the line to the load and back. The fact that the voltage drops to zero at $t = 9$ μs implies that the reflected wave is exactly equal to V_1^+ in magnitude, but opposite in polarity. That is,

$$V_1^- = -V_1^+.$$

This in turn implies that $\Gamma_L = -1$, which means that the load is a short circuit:

$$Z_L = 0.$$

(c) After V_1^- arrives at the generator end, it encounters a reflection coefficient Γ_g. The voltage at 15 μs is composed of:

$$\begin{aligned} V &= V_1^+ + V_1^- + V_2^+ \\ &= (1 + \Gamma_L + \Gamma_L \Gamma_g) V_1^+ \\ \frac{V}{V_1^+} &= 1 - 1 - \Gamma_g \end{aligned}$$

From the figure, $V/V_1^+ = -3/12 = -1/4$. Hence,

$$\Gamma_g = \frac{1}{4} \,,$$

which means that

$$R_g = \left(\frac{1+\Gamma_g}{1-\Gamma_g}\right) Z_0 = \left(\frac{1+0.25}{1-0.25}\right) 50 = 83.3\ \Omega.$$

(d)

$$V_1^+ = 12 = \frac{V_g Z_0}{R_g + Z_0}$$

$$V_g = \frac{12(R_g + Z_0)}{Z_0} = \frac{12(83.3+50)}{50} = 32\ \text{V}.$$