

Problem 2.9 A lossless microstrip line uses a 1-mm-wide conducting strip over a 1-cm-thick substrate with $\epsilon_r = 2.5$. Determine the line parameters, ϵ_{eff} , Z_0 , and β at 10 GHz. Compare your results with those obtained by using CD Module 2.3. Include a printout of the screen display.

Solution: Given

$$\begin{aligned}w &= 10^{-3} \text{ m}, \\h &= 10^{-2} \text{ m}, \\ \epsilon_r &= 2.5, \\f &= 1 \times 10^{10} \text{ Hz}, \\s &= \frac{w}{h} = 0.1.\end{aligned}$$

From Eq. (2.36),

$$\epsilon_{\text{eff}} = \frac{\epsilon_r + 1}{2} + \left(\frac{\epsilon_r - 1}{2} \right) \left(1 + \frac{10}{s} \right)^{-xy}$$

with

$$\begin{aligned}x &= 0.56 \left[\frac{\epsilon_r - 0.9}{\epsilon_r + 3} \right]^{0.05} \\&= 0.56 \left[\frac{2.5 - 0.9}{2.5 + 3} \right]^{0.05} = 0.526, \\y &= 1 + 0.02 \ln \left(\frac{s^4 + 3.7 \times 10^{-4} s^2}{s^4 + 0.43} \right) \\&\quad + 0.05 \ln(1 + 1.7 \times 10^{-4} s^3) \\&\approx 0.83,\end{aligned}$$

which leads to

$$\epsilon_{\text{eff}} \approx 1.85.$$

By Eq. (2.39),

$$Z_0 = \frac{60}{\sqrt{\epsilon_{\text{eff}}}} \ln \left\{ \frac{6 + (2\pi - 6)e^{-t}}{s} + \sqrt{1 + \frac{4}{s^2}} \right\}$$

with

$$t = \left(\frac{30.67}{s} \right)^{0.75} = \left(\frac{30.67}{0.1} \right)^{0.75} = 73.29.$$

Hence,

$$Z_0 = 193.3 \, \Omega.$$

Also,

$$\beta = \frac{\omega}{c} \sqrt{\epsilon_{\text{eff}}} = \frac{2\pi \times 10^{10}}{3 \times 10^8} \sqrt{1.85} = 284.87 \text{ rad/m.}$$

