

Problem 3.12 Two lines in the x - y plane are described by the expressions:

$$\text{Line 1} \quad x + 2y = -6,$$

$$\text{Line 2} \quad 3x + 4y = 8.$$

Use vector algebra to find the smaller angle between the lines at their intersection point.

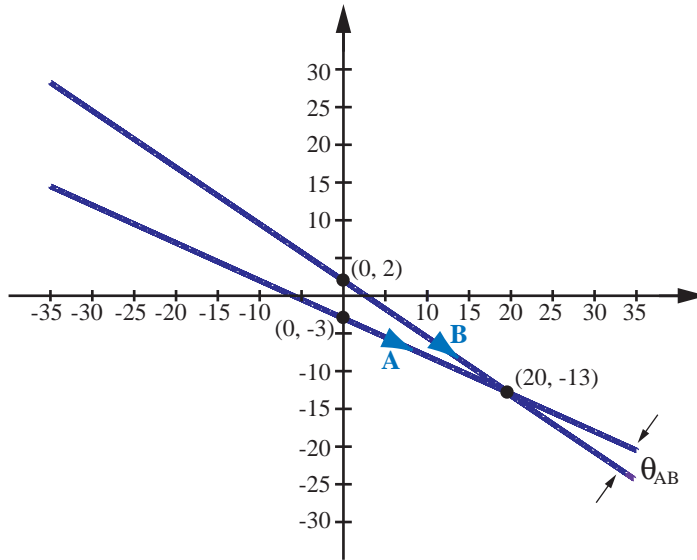


Figure P3.12: Lines 1 and 2.

Solution: Intersection point is found by solving the two equations simultaneously:

$$-2x - 4y = 12,$$

$$3x + 4y = 8.$$

The sum gives $x = 20$, which, when used in the first equation, gives $y = -13$.

Hence, intersection point is $(20, -13)$.

Another point on line 1 is $x = 0$, $y = -3$. Vector **A** from $(0, -3)$ to $(20, -13)$ is

$$\mathbf{A} = \hat{\mathbf{x}}(20) + \hat{\mathbf{y}}(-13 + 3) = \hat{\mathbf{x}}20 - \hat{\mathbf{y}}10,$$

$$|\mathbf{A}| = \sqrt{20^2 + 10^2} = \sqrt{500}.$$

A point on line 2 is $x = 0$, $y = 2$. Vector **B** from $(0, 2)$ to $(20, -13)$ is

$$\mathbf{B} = \hat{\mathbf{x}}(20) + \hat{\mathbf{y}}(-13 - 2) = \hat{\mathbf{x}}20 - \hat{\mathbf{y}}15,$$

$$|\mathbf{B}| = \sqrt{20^2 + 15^2} = \sqrt{625}.$$

Angle between \mathbf{A} and \mathbf{B} is

$$\theta_{AB} = \cos^{-1} \left(\frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{A}||\mathbf{B}|} \right) = \cos^{-1} \left(\frac{400 + 150}{\sqrt{500} \cdot \sqrt{625}} \right) = 10.3^\circ.$$
