

Problem 3.13 A given line is described by

$$x + 2y = 4.$$

Vector **A** starts at the origin and ends at point **P** on the line such that **A** is orthogonal to the line. Find an expression for **A**.

Solution: We first plot the given line. Next we find vector **B** which connects point $P_1 = (0, 2)$ to $P_2 = (4, 0)$, both of which are on the line:

$$\mathbf{B} = \hat{\mathbf{x}}(4 - 0) + \hat{\mathbf{y}}(0 - 2) = \hat{\mathbf{x}}4 - \hat{\mathbf{y}}2.$$

Vector **A** starts at the origin and ends on the line at **P**. If the x -coordinate of **P** is x ,

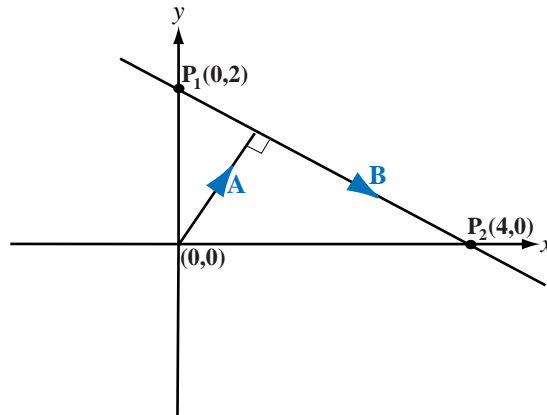


Figure P3.13: Given line and vector **A**.

then its y -coordinate has to be $(4 - x)/2$ in order to be on the line. Hence **P** is at $(x, (4 - x)/2)$. Vector **A** is

$$\mathbf{A} = \hat{\mathbf{x}}x + \hat{\mathbf{y}}\left(\frac{4 - x}{2}\right).$$

But **A** is perpendicular to the line. Hence,

$$\begin{aligned} \mathbf{A} \cdot \mathbf{B} &= 0, \\ \left[\hat{\mathbf{x}}x + \hat{\mathbf{y}}\left(\frac{4 - x}{2}\right) \right] \cdot (\hat{\mathbf{x}}4 - \hat{\mathbf{y}}2) &= 0, \\ 4x - (4 - x) &= 0, \quad \text{or} \\ x &= \frac{4}{5} = 0.8. \end{aligned}$$

Hence,

$$\mathbf{A} = \hat{\mathbf{x}}0.8 + \hat{\mathbf{y}}\left(\frac{4-0.8}{2}\right) = \hat{\mathbf{x}}0.8 + \hat{\mathbf{y}}1.6.$$
