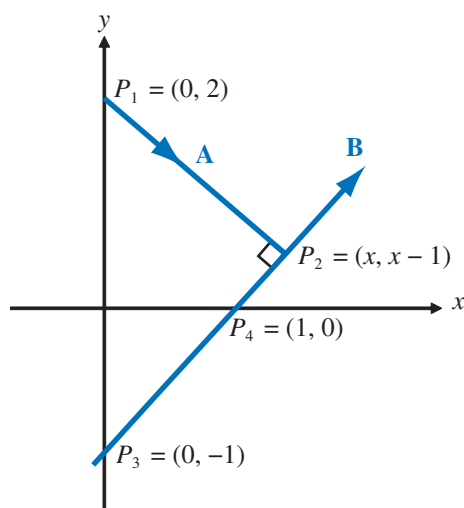


Problem 3.18 A given line is described by the equation:

$$y = x - 1.$$

Vector **A** starts at point $P_1 = (0, 2)$ and ends at point P_2 on the line such that **A** is orthogonal to the line. Find an expression for **A**.

Solution: We first plot the given line.



Next we find a vector **B** which connects point $P_3 = (0, -1)$ to point $P_4 = (1, 0)$, both of which are on the line. Hence,

$$\mathbf{B} = \hat{\mathbf{x}}(1 - 0) + \hat{\mathbf{y}}(0 + 1) = \hat{\mathbf{x}} + \hat{\mathbf{y}}.$$

Vector **A** starts at $P_1 = (0, 2)$ and ends on the line at P_2 . If the x -coordinate of P_2 is x , then its y -coordinate has to be $y = x - 1$, per the equation for the line. Thus, P_2 is at $(x, x - 1)$, and vector **A** is

$$\mathbf{A} = \hat{\mathbf{x}}(x - 0) + \hat{\mathbf{y}}(x - 1 - 2) = \hat{\mathbf{x}}x + \hat{\mathbf{y}}(x - 3).$$

Since **A** is orthogonal to **B**,

$$\mathbf{A} \cdot \mathbf{B} = 0,$$

$$[\hat{\mathbf{x}}x + \hat{\mathbf{y}}(x - 3)] \cdot (\hat{\mathbf{x}} + \hat{\mathbf{y}}) = 0$$

$$x + x - 3 = 0$$

$$x = \frac{3}{2}.$$

Finally,

$$\begin{aligned}\mathbf{A} &= \hat{\mathbf{x}}x + \hat{\mathbf{y}}(x-3) = \hat{\mathbf{x}}\frac{3}{2} + \hat{\mathbf{y}}\left(\frac{3}{2} - 3\right) \\ &= \hat{\mathbf{x}}\frac{3}{2} - \hat{\mathbf{y}}\frac{3}{2}.\end{aligned}$$
