

Problem 3.25 Use the appropriate expression for the differential surface area ds to determine the area of each of the following surfaces:

- (a) $r = 3$; $0 \leq \phi \leq \pi/3$; $-2 \leq z \leq 2$,
- (b) $2 \leq r \leq 5$; $\pi/2 \leq \phi \leq \pi$; $z = 0$,
- (c) $2 \leq r \leq 5$; $\phi = \pi/4$; $-2 \leq z \leq 2$,
- (d) $R = 2$; $0 \leq \theta \leq \pi/3$; $0 \leq \phi \leq \pi$,
- (e) $0 \leq R \leq 5$; $\theta = \pi/3$; $0 \leq \phi \leq 2\pi$.

Also sketch the outlines of each of the surfaces.

Solution:

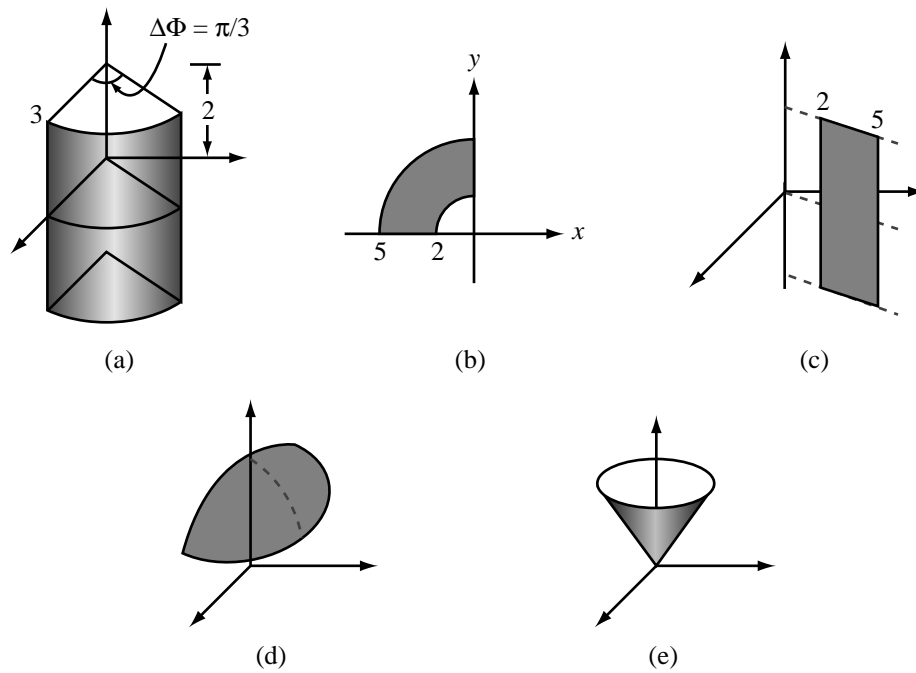


Figure P3.25: Surfaces described by Problem 3.25.

(a) Using Eq. (3.43a),

$$A = \int_{z=-2}^2 \int_{\phi=0}^{\pi/3} (r)|_{r=3} d\phi dz = \left((3\phi z) \Big|_{\phi=0}^{\pi/3} \right) \Big|_{z=-2}^2 = 4\pi.$$

(b) Using Eq. (3.43c),

$$A = \int_{r=2}^5 \int_{\phi=\pi/2}^{\pi} (r)|_{z=0} d\phi dr = \left(\left(\frac{1}{2} r^2 \phi \right) \Big|_{r=2}^5 \right) \Big|_{\phi=\pi/2}^{\pi} = \frac{21\pi}{4}.$$

(c) Using Eq. (3.43b),

$$A = \int_{z=-2}^2 \int_{r=2}^5 (1)|_{\phi=\pi/4} dr dz = \left((rz)|_{z=-2}^2 \right) \Big|_{r=2}^5 = 12.$$

(d) Using Eq. (3.50b),

$$A = \int_{\theta=0}^{\pi/3} \int_{\phi=0}^{\pi} (R^2 \sin \theta)|_{R=2} d\phi d\theta = \left((-4\phi \cos \theta)|_{\theta=0}^{\pi/3} \right) \Big|_{\phi=0}^{\pi} = 2\pi.$$

(e) Using Eq. (3.50c),

$$A = \int_{R=0}^5 \int_{\phi=0}^{2\pi} (R \sin \theta)|_{\theta=\pi/3} d\phi dR = \left(\left(\frac{1}{2} R^2 \phi \sin \frac{\pi}{3} \right) \Big|_{\phi=0}^{2\pi} \right) \Big|_{R=0}^5 = \frac{25\sqrt{3}\pi}{2}.$$
