

Problem 3.27 A section of a sphere is described by $0 \leq R \leq 2$, $0 \leq \theta \leq 90^\circ$, and $30^\circ \leq \phi \leq 90^\circ$. Find:

- (a) the surface area of the spherical section,
- (b) the enclosed volume.

Also sketch the outline of the section.

Solution:

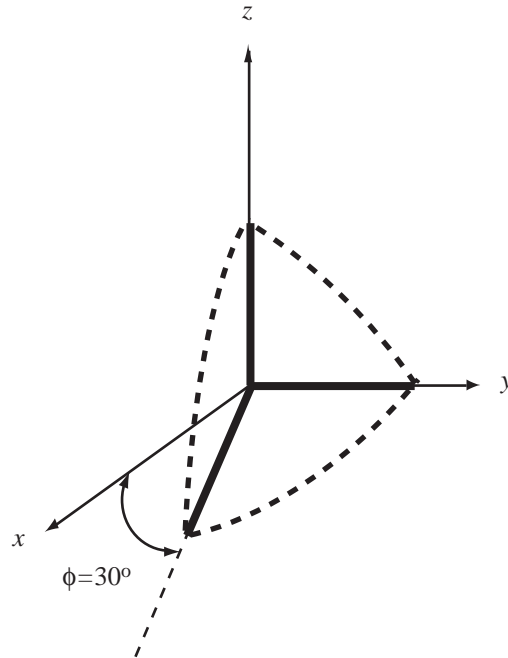


Figure P3.27: Outline of section.

$$\begin{aligned}
 S &= \int_{\phi=\pi/6}^{\pi/2} \int_{\theta=0}^{\pi/2} R^2 \sin \theta \, d\theta \, d\phi \Big|_{R=2} \\
 &= 4 \left(\frac{\pi}{2} - \frac{\pi}{6} \right) \left[-\cos \theta \Big|_0^{\pi/2} \right] = 4 \times \frac{\pi}{3} = \frac{4\pi}{3} \quad (\text{m}^2), \\
 V &= \int_{R=0}^2 \int_{\phi=\pi/6}^{\pi/2} \int_{\theta=0}^{\pi/2} R^2 \sin \theta \, dR \, d\theta \, d\phi \\
 &= \frac{R^3}{3} \Big|_0^2 \left(\frac{\pi}{2} - \frac{\pi}{6} \right) \left[-\cos \theta \Big|_0^{\pi/2} \right] = \frac{8}{3} \frac{\pi}{3} = \frac{8\pi}{9} \quad (\text{m}^3).
 \end{aligned}$$
