Problem 3.28 A vector field is given in cylindrical coordinates by

$$\mathbf{E} = \hat{\mathbf{r}} r\cos\phi + \hat{\boldsymbol{\phi}} r\sin\phi + \hat{\mathbf{z}} z^2.$$

Point $P = (2, \pi, 3)$ is located on the surface of the cylinder described by $r = 2$. At point P, find:

(a) the vector component of $\mathbf{E}$ perpendicular to the cylinder,

(b) the vector component of $\mathbf{E}$ tangential to the cylinder.

Solution:

(a) $\mathbf{E}_{\mathrm{n}} = \hat{\mathbf{r}}(\hat{\mathbf{r}} \cdot \mathbf{E}) = \hat{\mathbf{r}}[\hat{\mathbf{r}} \cdot (\hat{\mathbf{r}} r\cos\phi + \hat{\boldsymbol{\phi}} r\sin\phi + \hat{\mathbf{z}} z^2)] = \hat{\mathbf{r}} r\cos\phi.$

At $P = (2, \pi, 3)$, $\mathbf{E}_{\mathrm{n}} = \hat{\mathbf{r}} 2\cos\pi = -\hat{\mathbf{r}} 2.$

(b) $\mathbf{E}_{\mathrm{t}} = \mathbf{E} - \mathbf{E}_{\mathrm{n}} = \hat{\boldsymbol{\phi}} r\sin\phi + \hat{\mathbf{z}} z^2.$

At $P = (2, \pi, 3)$, $\mathbf{E}_{\mathrm{t}} = \hat{\boldsymbol{\phi}} 2\sin\pi + \hat{\mathbf{z}} 3^2 = \hat{\mathbf{z}} 9.$