

**Problem 3.29** At a given point in space, vectors **A** and **B** are given in spherical coordinates by

$$\begin{aligned}\mathbf{A} &= \hat{\mathbf{R}}4 + \hat{\boldsymbol{\theta}}2 - \hat{\boldsymbol{\phi}}, \\ \mathbf{B} &= -\hat{\mathbf{R}}2 + \hat{\boldsymbol{\phi}}3.\end{aligned}$$

Find:

- (a) the scalar component, or projection, of **B** in the direction of **A**,
- (b) the vector component of **B** in the direction of **A**,
- (c) the vector component of **B** perpendicular to **A**.

**Solution:**

- (a) Scalar component of **B** in direction of **A**:

$$\begin{aligned}C = \mathbf{B} \cdot \hat{\mathbf{a}} &= \mathbf{B} \cdot \frac{\mathbf{A}}{|\mathbf{A}|} = (-\hat{\mathbf{R}}2 + \hat{\boldsymbol{\phi}}3) \cdot \frac{(\hat{\mathbf{R}}4 + \hat{\boldsymbol{\theta}}2 - \hat{\boldsymbol{\phi}})}{\sqrt{16 + 4 + 1}} \\ &= \frac{-8 - 3}{\sqrt{21}} = -\frac{11}{\sqrt{21}} = -2.4.\end{aligned}$$

- (b) Vector component of **B** in direction of **A**:

$$\begin{aligned}\mathbf{C} = \hat{\mathbf{a}}C &= \mathbf{A} \frac{C}{|\mathbf{A}|} = (\hat{\mathbf{R}}4 + \hat{\boldsymbol{\theta}}2 - \hat{\boldsymbol{\phi}}) \frac{(-2.4)}{\sqrt{21}} \\ &= -(\hat{\mathbf{R}}2.09 + \hat{\boldsymbol{\theta}}1.05 - \hat{\boldsymbol{\phi}}0.52).\end{aligned}$$

- (c) Vector component of **B** perpendicular to **A**:

$$\begin{aligned}\mathbf{D} = \mathbf{B} - \mathbf{C} &= (-\hat{\mathbf{R}}2 + \hat{\boldsymbol{\phi}}3) + (\hat{\mathbf{R}}2.09 + \hat{\boldsymbol{\theta}}1.05 - \hat{\boldsymbol{\phi}}0.52) \\ &= \hat{\mathbf{R}}0.09 + \hat{\boldsymbol{\theta}}1.05 + \hat{\boldsymbol{\phi}}2.48.\end{aligned}$$


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