Problem 3.3 In Cartesian coordinates, the three corners of a triangle are $P_1 = (0,4,4)$, $P_2 = (4,-4,4)$, and $P_3 = (2,2,-4)$. Find the area of the triangle.

Solution: Let $\mathbf{B} = \overrightarrow{P_1P_2} = \hat{\mathbf{x}}4 - \hat{\mathbf{y}}8$ and $\mathbf{C} = \overrightarrow{P_1P_3} = \hat{\mathbf{x}}2 - \hat{\mathbf{y}}2 - \hat{\mathbf{z}}8$ represent two sides of the triangle. Since the magnitude of the cross product is the area of the parallelogram (see the definition of cross product in Section 3-1.4), half of this is the area of the triangle:

$$\begin{aligned} A = \tfrac{1}{2}|\mathbf{B}\times\mathbf{C}| &= \tfrac{1}{2}|(\hat{\mathbf{x}}4 - \hat{\mathbf{y}}8)\times(\hat{\mathbf{x}}2 - \hat{\mathbf{y}}2 - \hat{\mathbf{z}}8)| \\ &= \tfrac{1}{2}|\hat{\mathbf{x}}(-8)(-8) + \hat{\mathbf{y}}(-(4)(-8)) + \hat{\mathbf{z}}(4(-2) - (-8)2)| \\ &= \tfrac{1}{2}|\hat{\mathbf{x}}64 + \hat{\mathbf{y}}32 + \hat{\mathbf{z}}8| = \tfrac{1}{2}\sqrt{64^2 + 32^2 + 8^2} = \tfrac{1}{2}\sqrt{5184} = 36, \end{aligned}$$

where the cross product is evaluated with Eq. (3.27).