Problem 3.30 Given vectors

$$\mathbf{A} = \hat{\mathbf{r}}(\cos\phi + 3z) - \hat{\boldsymbol{\phi}}(2r + 4\sin\phi) + \hat{\mathbf{z}}(r - 2z),$$
$$\mathbf{B} = -\hat{\mathbf{r}}\sin\phi + \hat{\mathbf{z}}\cos\phi,$$

find

(a) θ_{AB} at $(2, \pi/2, 0)$,

(b) a unit vector perpendicular to both **A** and **B** at $(2, \pi/3, 1)$.

Solution: It doesn't matter whether the vectors are evaluated before vector products are calculated, or if the vector products are directly calculated and the general results are evaluated at the specific point in question.

(a) At $(2, \pi/2, 0)$, $\mathbf{A} = -\hat{\boldsymbol{\phi}}8 + \hat{\mathbf{z}}2$ and $\mathbf{B} = -\hat{\mathbf{r}}$. From Eq. (3.18),

$$\theta_{AB} = \cos^{-1}\left(\frac{\mathbf{A}\cdot\mathbf{B}}{AB}\right) = \cos^{-1}\left(\frac{0}{AB}\right) = 90^\circ.$$

(b) At $(2, \pi/3, 1)$, $\mathbf{A} = \hat{\mathbf{r}}\frac{7}{2} - \hat{\boldsymbol{\phi}}4(1 + \frac{1}{2}\sqrt{3})$ and $\mathbf{B} = -\hat{\mathbf{r}}\frac{1}{2}\sqrt{3} + \hat{\mathbf{z}}\frac{1}{2}$. Since $\mathbf{A}\times\mathbf{B}$ is perpendicular to both **A** and **B**, a unit vector perpendicular to both **A** and **B** is given by

$$\pm\frac{\mathbf{A}\times\mathbf{B}}{|\mathbf{A}\times\mathbf{B}|} = \pm\frac{\hat{\mathbf{r}}(-4(1+\frac{1}{2}\sqrt{3}))(\frac{1}{2}) - \hat{\boldsymbol{\phi}}(\frac{7}{2})(\frac{1}{2}) - \hat{\mathbf{z}}(4(1+\frac{1}{2}\sqrt{3}))(\frac{1}{2}\sqrt{3})}{\sqrt{(2(1+\frac{1}{2}\sqrt{3}))^2 + (\frac{7}{4})^2 + (3+2\sqrt{3})^2}}$$
$$\approx \mp(\hat{\mathbf{r}}0.487 + \hat{\boldsymbol{\phi}}0.228 + \hat{\mathbf{z}}0.843).$$