

Problem 3.34 Transform the following vectors into cylindrical coordinates and then evaluate them at the indicated points:

- (a) $\mathbf{A} = \hat{\mathbf{x}}(x+y)$ at $P_1 = (1, 2, 3)$,
- (b) $\mathbf{B} = \hat{\mathbf{x}}(y-x) + \hat{\mathbf{y}}(x-y)$ at $P_2 = (1, 0, 2)$,
- (c) $\mathbf{C} = \hat{\mathbf{x}}y^2/(x^2+y^2) - \hat{\mathbf{y}}x^2/(x^2+y^2) + \hat{\mathbf{z}}4$ at $P_3 = (1, -1, 2)$,
- (d) $\mathbf{D} = \hat{\mathbf{R}} \sin \theta + \hat{\boldsymbol{\theta}} \cos \theta + \hat{\boldsymbol{\phi}} \cos^2 \phi$ at $P_4(2, \pi/2, \pi/4)$,
- (e) $\mathbf{E} = \hat{\mathbf{R}} \cos \phi + \hat{\boldsymbol{\theta}} \sin \phi + \hat{\boldsymbol{\phi}} \sin^2 \theta$ at $P_5 = (3, \pi/2, \pi)$.

Solution: From Table 3-2:

(a)

$$\begin{aligned}\mathbf{A} &= (\hat{\mathbf{r}} \cos \phi - \hat{\boldsymbol{\phi}} \sin \phi)(r \cos \phi + r \sin \phi) \\ &= \hat{\mathbf{r}} r \cos \phi (\cos \phi + \sin \phi) - \hat{\boldsymbol{\phi}} r \sin \phi (\cos \phi + \sin \phi), \\ P_1 &= (\sqrt{1^2 + 2^2}, \tan^{-1}(2/1), 3) = (\sqrt{5}, 63.4^\circ, 3), \\ \mathbf{A}(P_1) &= (\hat{\mathbf{r}} 0.447 - \hat{\boldsymbol{\phi}} 0.894) \sqrt{5} (.447 + .894) = \hat{\mathbf{r}} 1.34 - \hat{\boldsymbol{\phi}} 2.68.\end{aligned}$$

(b)

$$\begin{aligned}\mathbf{B} &= (\hat{\mathbf{r}} \cos \phi - \hat{\boldsymbol{\phi}} \sin \phi)(r \sin \phi - r \cos \phi) + (\hat{\boldsymbol{\phi}} \cos \phi + \hat{\mathbf{r}} \sin \phi)(r \cos \phi - r \sin \phi) \\ &= \hat{\mathbf{r}} r (2 \sin \phi \cos \phi - 1) + \hat{\boldsymbol{\phi}} r (\cos^2 \phi - \sin^2 \phi) = \hat{\mathbf{r}} r (\sin 2\phi - 1) + \hat{\boldsymbol{\phi}} r \cos 2\phi, \\ P_2 &= (\sqrt{1^2 + 0^2}, \tan^{-1}(0/1), 2) = (1, 0^\circ, 2), \\ \mathbf{B}(P_2) &= -\hat{\mathbf{r}} + \hat{\boldsymbol{\phi}}.\end{aligned}$$

(c)

$$\begin{aligned}\mathbf{C} &= (\hat{\mathbf{r}} \cos \phi - \hat{\boldsymbol{\phi}} \sin \phi) \frac{r^2 \sin^2 \phi}{r^2} - (\hat{\boldsymbol{\phi}} \cos \phi + \hat{\mathbf{r}} \sin \phi) \frac{r^2 \cos^2 \phi}{r^2} + \hat{\mathbf{z}} 4 \\ &= \hat{\mathbf{r}} \sin \phi \cos \phi (\sin \phi - \cos \phi) - \hat{\boldsymbol{\phi}} (\sin^3 \phi + \cos^3 \phi) + \hat{\mathbf{z}} 4, \\ P_3 &= (\sqrt{1^2 + (-1)^2}, \tan^{-1}(-1/1), 2) = (\sqrt{2}, -45^\circ, 2), \\ \mathbf{C}(P_3) &= \hat{\mathbf{r}} 0.707 + \hat{\mathbf{z}} 4.\end{aligned}$$

(d)

$$\begin{aligned}\mathbf{D} &= (\hat{\mathbf{r}} \sin \theta + \hat{\mathbf{z}} \cos \theta) \sin \theta + (\hat{\mathbf{r}} \cos \theta - \hat{\mathbf{z}} \sin \theta) \cos \theta + \hat{\boldsymbol{\phi}} \cos^2 \phi = \hat{\mathbf{r}} + \hat{\boldsymbol{\phi}} \cos^2 \phi, \\ P_4 &= (2 \sin(\pi/2), \pi/4, 2 \cos(\pi/2)) = (2, 45^\circ, 0), \\ \mathbf{D}(P_4) &= \hat{\mathbf{r}} + \hat{\boldsymbol{\phi}} \frac{1}{2}.\end{aligned}$$

(e)

$$\mathbf{E} = (\hat{\mathbf{r}} \sin \theta + \hat{\mathbf{z}} \cos \theta) \cos \phi + (\hat{\mathbf{r}} \cos \theta - \hat{\mathbf{z}} \sin \theta) \sin \phi + \hat{\boldsymbol{\phi}} \sin^2 \theta,$$

$$P_5 = \left(3, \frac{\pi}{2}, \pi\right),$$

$$\mathbf{E}(P_5) = \left(\hat{\mathbf{r}} \sin \frac{\pi}{2} + \hat{\mathbf{z}} \cos \frac{\pi}{2}\right) \cos \pi + \left(\hat{\mathbf{r}} \cos \frac{\pi}{2} - \hat{\mathbf{z}} \sin \frac{\pi}{2}\right) \sin \pi + \hat{\boldsymbol{\phi}} \sin^2 \frac{\pi}{2} = -\hat{\mathbf{r}} + \hat{\boldsymbol{\phi}}.$$
