

Problem 3.36 Find the gradient of the following scalar functions:

- (a) $T = 3/(x^2 + z^2)$,
- (b) $V = xy^2z^4$,
- (c) $U = z \cos \phi / (1 + r^2)$,
- (d) $W = e^{-R} \sin \theta$,
- (e) $S = 4x^2e^{-z} + y^3$,
- (f) $N = r^2 \cos^2 \phi$,
- (g) $M = R \cos \theta \sin \phi$.

Solution:

(a) From Eq. (3.72),

$$\nabla T = -\hat{\mathbf{x}} \frac{6x}{(x^2 + z^2)^2} - \hat{\mathbf{z}} \frac{6z}{(x^2 + z^2)^2}.$$

(b) From Eq. (3.72),

$$\nabla V = \hat{\mathbf{x}} y^2 z^4 + \hat{\mathbf{y}} 2xyz^4 + \hat{\mathbf{z}} 4xy^2 z^3.$$

(c) From Eq. (3.82),

$$\nabla U = -\hat{\mathbf{r}} \frac{2rz \cos \phi}{(1 + r^2)^2} - \hat{\boldsymbol{\phi}} \frac{z \sin \phi}{r(1 + r^2)} + \hat{\mathbf{z}} \frac{\cos \phi}{1 + r^2}.$$

(d) From Eq. (3.83),

$$\nabla W = -\hat{\mathbf{R}} e^{-R} \sin \theta + \hat{\boldsymbol{\theta}} (e^{-R}/R) \cos \theta.$$

(e) From Eq. (3.72),

$$\begin{aligned} S &= 4x^2 e^{-z} + y^3, \\ \nabla S &= \hat{\mathbf{x}} \frac{\partial S}{\partial x} + \hat{\mathbf{y}} \frac{\partial S}{\partial y} + \hat{\mathbf{z}} \frac{\partial S}{\partial z} = \hat{\mathbf{x}} 8x e^{-z} + \hat{\mathbf{y}} 3y^2 - \hat{\mathbf{z}} 4x^2 e^{-z}. \end{aligned}$$

(f) From Eq. (3.82),

$$\begin{aligned} N &= r^2 \cos^2 \phi, \\ \nabla N &= \hat{\mathbf{r}} \frac{\partial N}{\partial r} + \hat{\boldsymbol{\phi}} \frac{1}{r} \frac{\partial N}{\partial \phi} + \hat{\mathbf{z}} \frac{\partial N}{\partial z} = \hat{\mathbf{r}} 2r \cos^2 \phi - \hat{\boldsymbol{\phi}} 2r \sin \phi \cos \phi. \end{aligned}$$

(g) From Eq. (3.83),

$$\begin{aligned} M &= R \cos \theta \sin \phi, \\ \nabla M &= \hat{\mathbf{R}} \frac{\partial M}{\partial R} + \hat{\boldsymbol{\theta}} \frac{1}{R} \frac{\partial M}{\partial \theta} + \hat{\boldsymbol{\phi}} \frac{1}{R \sin \theta} \frac{\partial M}{\partial \phi} = \hat{\mathbf{R}} \cos \theta \sin \phi - \hat{\boldsymbol{\theta}} \sin \theta \sin \phi + \hat{\boldsymbol{\phi}} \frac{\cos \phi}{\tan \theta}. \end{aligned}$$
