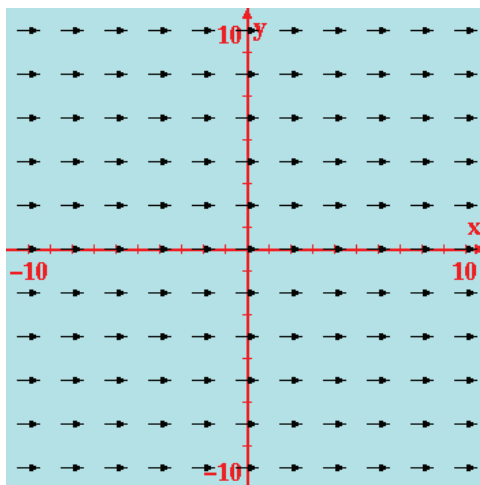


**Problem 3.37** For each of the following scalar fields, obtain an analytical solution for  $\nabla T$  and generate a corresponding arrow representation.

- (a)  $T = 10 + x$ , for  $-10 \leq x \leq 10$
- (b)  $T = x^2$ , for  $-10 \leq x \leq 10$
- (c)  $T = 100 + xy$ , for  $-10 \leq x \leq 10$
- (d)  $T = x^2y^2$ , for  $-10 \leq x, y \leq 10$
- (e)  $T = 20 + x + y$ , for  $-10 \leq x, y \leq 10$
- (f)  $T = 1 + \sin(\pi x/3)$ , for  $-10 \leq x \leq 10$
- (g)  $T = 1 + \cos(\pi x/3)$ , for  $-10 \leq x \leq 10$
- (h)  $T = 15 + r \cos \phi$ , for  $\begin{cases} 0 \leq r \leq 10 \\ 0 \leq \phi \leq 2\pi. \end{cases}$
- (i)  $T = 15 + r \cos^2 \phi$ , for  $\begin{cases} 0 \leq r \leq 10 \\ 0 \leq \phi \leq 2\pi. \end{cases}$

**Solution:**

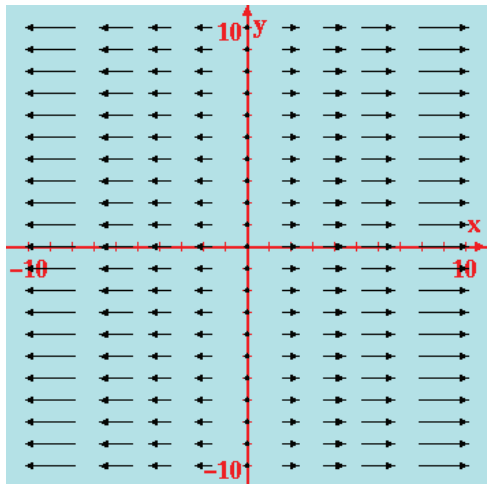
(a)



$$\begin{aligned}
 T &= 10 + x \\
 \nabla T &= \hat{\mathbf{x}} \frac{\partial T}{\partial x} + \hat{\mathbf{y}} \frac{\partial T}{\partial y} + \hat{\mathbf{z}} \frac{\partial T}{\partial z} \\
 &= \hat{\mathbf{x}}.
 \end{aligned}$$

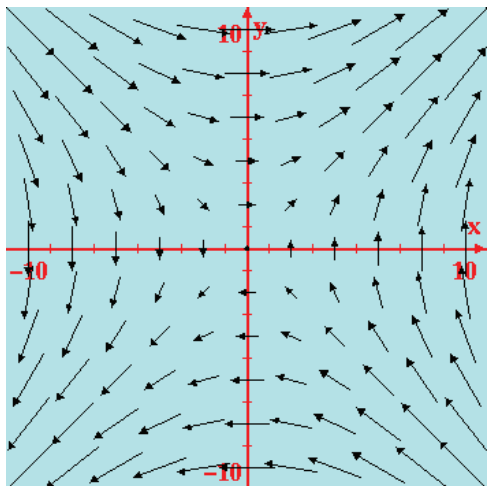
The direction of  $\nabla T$  displays the fact that  $T$  increases linearly with  $x$  only.

(b)



$$\begin{aligned} T &= x^2 \\ \nabla T &= \hat{\mathbf{x}} \frac{\partial T}{\partial x} + \hat{\mathbf{y}} \frac{\partial T}{\partial y} + \hat{\mathbf{z}} \frac{\partial T}{\partial z} \\ &= \hat{\mathbf{x}} 2x. \end{aligned}$$

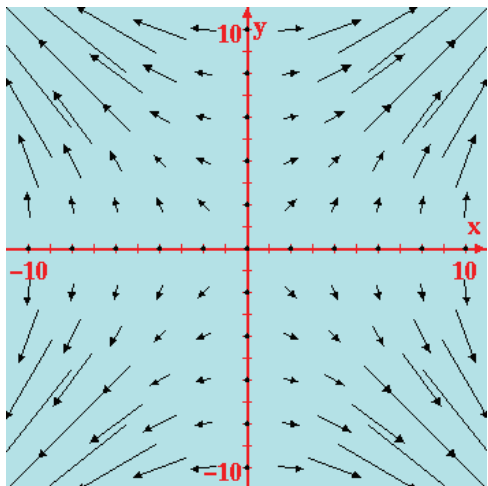
(c)



$$\begin{aligned} T &= 100 + xy \\ \nabla T &= \hat{\mathbf{x}} \frac{\partial T}{\partial x} + \hat{\mathbf{y}} \frac{\partial T}{\partial y} + \hat{\mathbf{z}} \frac{\partial T}{\partial z} \\ &= \hat{\mathbf{x}} y + \hat{\mathbf{y}} x. \end{aligned}$$

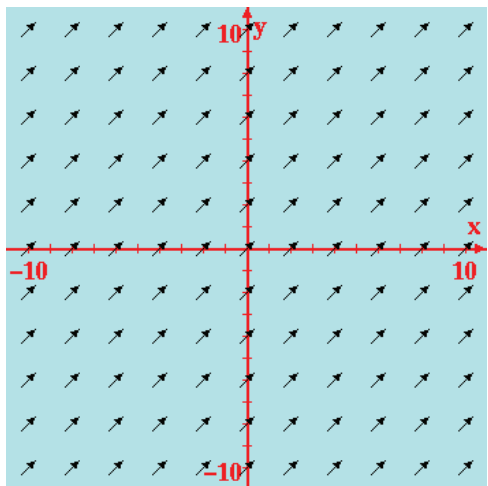
The magnitude of the gradient increases monotonically with  $x$ .

(d)



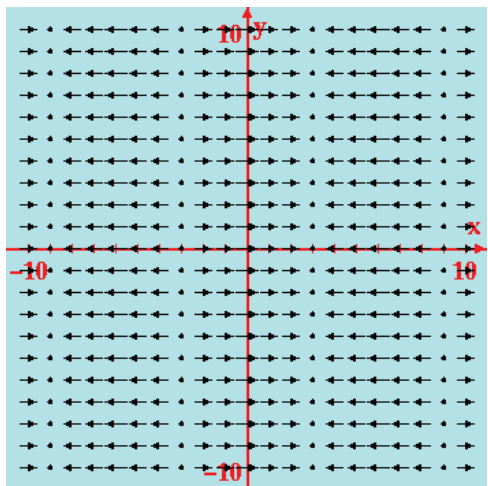
$$\begin{aligned} T &= x^2y^2 \\ \nabla T &= \hat{\mathbf{x}} \frac{\partial T}{\partial x} + \hat{\mathbf{y}} \frac{\partial T}{\partial y} + \hat{\mathbf{z}} \frac{\partial T}{\partial z} \\ &= \hat{\mathbf{x}} 2xy^2 + \hat{\mathbf{y}} 2x^2y. \end{aligned}$$

(e)



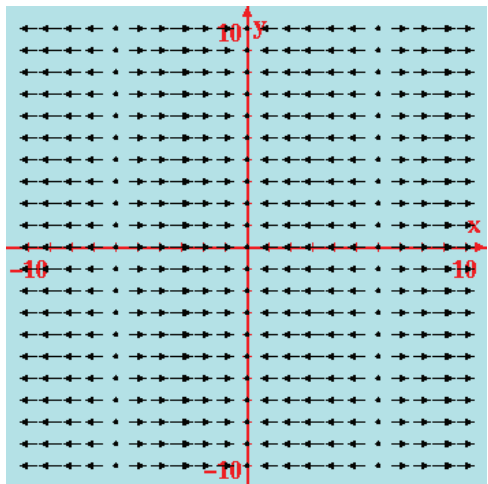
$$\begin{aligned} T &= 20 + x + y \\ \nabla T &= \hat{\mathbf{x}} \frac{\partial T}{\partial x} + \hat{\mathbf{y}} \frac{\partial T}{\partial y} + \hat{\mathbf{z}} \frac{\partial T}{\partial z} \\ &= \hat{\mathbf{x}} + \hat{\mathbf{y}}. \end{aligned}$$

(f)



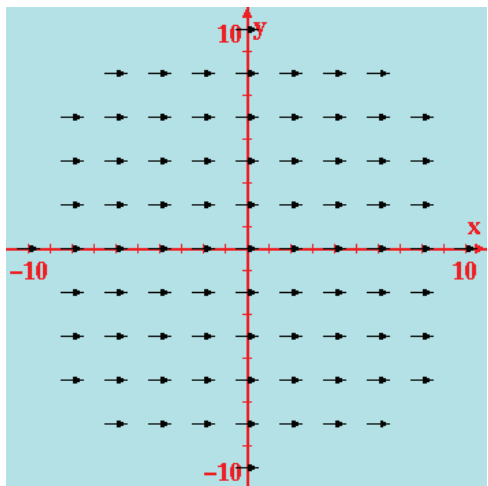
$$\begin{aligned} T &= 1 + \sin(\pi x/3) \\ \nabla T &= \hat{\mathbf{x}} \frac{\partial T}{\partial x} + \hat{\mathbf{y}} \frac{\partial T}{\partial y} + \hat{\mathbf{z}} \frac{\partial T}{\partial z} \\ &= \hat{\mathbf{x}} \frac{2\pi}{6} \cos(\pi x/3). \end{aligned}$$

(g)



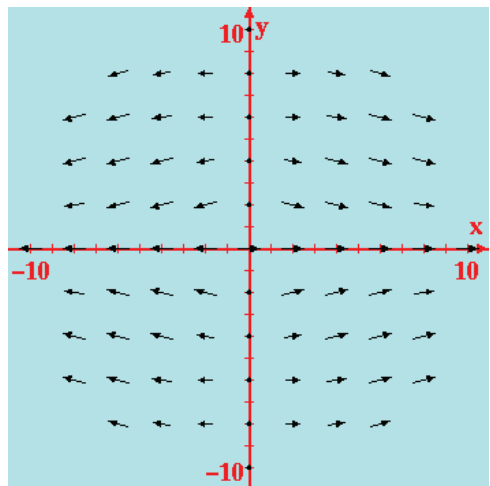
$$\begin{aligned} T &= 1 + \cos(\pi x/3) \\ \nabla T &= \hat{\mathbf{x}} \frac{\partial T}{\partial x} + \hat{\mathbf{y}} \frac{\partial T}{\partial y} + \hat{\mathbf{z}} \frac{\partial T}{\partial z} \\ &= -\hat{\mathbf{x}} \frac{2\pi}{6} \sin(\pi x/3). \end{aligned}$$

(h)



$$\begin{aligned} T &= 15 + r \cos \theta \\ \nabla T &= \hat{\mathbf{r}} \frac{\partial T}{\partial r} + \hat{\boldsymbol{\phi}} \frac{1}{r} \frac{\partial T}{\partial \phi} \\ &= \hat{\mathbf{r}} \cos \phi - \hat{\boldsymbol{\phi}} \sin \phi \\ &= \hat{\mathbf{x}}. \end{aligned}$$

(i)



$$\begin{aligned} T &= 15 + r \cos^2 \theta \\ \nabla T &= \hat{\mathbf{r}} \frac{\partial T}{\partial r} + \hat{\boldsymbol{\phi}} \frac{1}{r} \frac{\partial T}{\partial \phi} \\ &= \hat{\mathbf{r}} \cos^2 \phi - \hat{\boldsymbol{\phi}} 2 \sin \phi \cos \phi. \end{aligned}$$

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