

**Problem 3.38** The gradient of a scalar function  $T$  is given by

$$\nabla T = \hat{\mathbf{z}}e^{-3z}.$$

If  $T = 10$  at  $z = 0$ , find  $T(z)$ .

**Solution:**

$$\nabla T = \hat{\mathbf{z}}e^{-3z}.$$

By choosing  $P_1$  at  $z = 0$  and  $P_2$  at any point  $z$ , (3.76) becomes

$$\begin{aligned} T(z) - T(0) &= \int_0^z \nabla T \cdot d\mathbf{l}' = \int_0^z \hat{\mathbf{z}}e^{-3z'} \cdot (\hat{\mathbf{x}}dx' + \hat{\mathbf{y}}dy' + \hat{\mathbf{z}}dz') \\ &= \int_0^z e^{-3z'} dz' = -\frac{e^{-3z'}}{3} \Big|_0^z = \frac{1}{3}(1 - e^{-3z}). \end{aligned}$$

Hence,

$$T(z) = T(0) + \frac{1}{3}(1 - e^{-3z}) = 10 + \frac{1}{3}(1 - e^{-3z}).$$

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