

Problem 3.4 Given $\mathbf{A} = \hat{x}2 - \hat{y}3 + \hat{z}1$ and $\mathbf{B} = \hat{x}B_x + \hat{y}2 + \hat{z}B_z$:

- (a) find B_x and B_z if \mathbf{A} is parallel to \mathbf{B} ;
- (b) find a relation between B_x and B_z if \mathbf{A} is perpendicular to \mathbf{B} .

Solution:

- (a) If \mathbf{A} is parallel to \mathbf{B} , then their directions are equal or opposite: $\hat{\mathbf{a}}_A = \pm \hat{\mathbf{a}}_B$, or

$$\frac{\mathbf{A}}{|\mathbf{A}|} = \pm \frac{\mathbf{B}}{|\mathbf{B}|},$$

$$\frac{\hat{x}2 - \hat{y}3 + \hat{z}}{\sqrt{14}} = \pm \frac{\hat{x}B_x + \hat{y}2 + \hat{z}B_z}{\sqrt{4 + B_x^2 + B_z^2}}.$$

From the y-component,

$$\frac{-3}{\sqrt{14}} = \frac{\pm 2}{\sqrt{4 + B_x^2 + B_z^2}}$$

which can only be solved for the minus sign (which means that \mathbf{A} and \mathbf{B} must point in opposite directions for them to be parallel). Solving for $B_x^2 + B_z^2$,

$$B_x^2 + B_z^2 = \left(\frac{-2}{-3} \sqrt{14} \right)^2 - 4 = \frac{20}{9}.$$

From the x-component,

$$\frac{2}{\sqrt{14}} = \frac{-B_x}{\sqrt{56/9}}, \quad B_x = \frac{-2\sqrt{56}}{3\sqrt{14}} = \frac{-4}{3}$$

and, from the z-component,

$$B_z = \frac{-2}{3}.$$

This is consistent with our result for $B_x^2 + B_z^2$.

These results could also have been obtained by assuming θ_{AB} was 0° or 180° and solving $|\mathbf{A}||\mathbf{B}| = \pm \mathbf{A} \cdot \mathbf{B}$, or by solving $\mathbf{A} \times \mathbf{B} = 0$.

- (b) If \mathbf{A} is perpendicular to \mathbf{B} , then their dot product is zero (see Section 3-1.4). Using Eq. (3.21),

$$0 = \mathbf{A} \cdot \mathbf{B} = 2B_x - 6 + B_z,$$

or

$$B_z = 6 - 2B_x.$$

There are an infinite number of vectors which could be \mathbf{B} and be perpendicular to \mathbf{A} , but their x- and z-components must satisfy this relation.

This result could have also been obtained by assuming $\theta_{AB} = 90^\circ$ and calculating $|\mathbf{A}||\mathbf{B}| = |\mathbf{A} \times \mathbf{B}|$.
