

Problem 3.40 For the scalar function $V = xy^2 - z^2$, determine its directional derivative along the direction of vector $\mathbf{A} = (\hat{\mathbf{x}} - \hat{\mathbf{y}}z)$ and then evaluate it at $P = (1, -1, 4)$.

Solution: The directional derivative is given by Eq. (3.75) as $dV/dl = \nabla V \cdot \hat{\mathbf{a}}_l$, where the unit vector in the direction of \mathbf{A} is given by Eq. (3.2):

$$\hat{\mathbf{a}}_l = \frac{\hat{\mathbf{x}} - \hat{\mathbf{y}}z}{\sqrt{1 + z^2}},$$

and the gradient of V in Cartesian coordinates is given by Eq. (3.72):

$$\nabla V = \hat{\mathbf{x}}y^2 + \hat{\mathbf{y}}2xy - \hat{\mathbf{z}}2z.$$

Therefore, by Eq. (3.75),

$$\frac{dV}{dl} = \frac{y^2 - 2xyz}{\sqrt{1 + z^2}}.$$

At $P = (1, -1, 4)$,

$$\left(\frac{dV}{dl} \right) \Big|_{(1, -1, 4)} = \frac{9}{\sqrt{17}} = 2.18.$$
