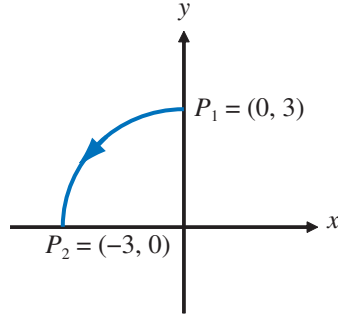


Problem 3.41 Evaluate the line integral of $\mathbf{E} = \hat{\mathbf{x}}x - \hat{\mathbf{y}}y$ along the segment P_1 to P_2 of the circular path shown in the figure.



Solution: We need to calculate:

$$\int_{P_1}^{P_2} \mathbf{E} \cdot d\boldsymbol{\ell}.$$

Since the path is along the perimeter of a circle, it is best to use cylindrical coordinates, which requires expressing both \mathbf{E} and $d\boldsymbol{\ell}$ in cylindrical coordinates. Using Table 3-2,

$$\begin{aligned} \mathbf{E} = \hat{\mathbf{x}}x - \hat{\mathbf{y}}y &= (\hat{\mathbf{r}} \cos \phi - \hat{\boldsymbol{\phi}} \sin \phi)r \cos \phi - (\hat{\mathbf{r}} \sin \phi + \hat{\boldsymbol{\phi}} \cos \phi)r \sin \phi \\ &= \hat{\mathbf{r}} r(\cos^2 \phi - \sin^2 \phi) - \hat{\boldsymbol{\phi}} 2r \sin \phi \cos \phi \end{aligned}$$

The designated path is along the ϕ -direction at a constant $r = 3$. From Table 3-1, the applicable component of $d\boldsymbol{\ell}$ is:

$$d\boldsymbol{\ell} = \hat{\boldsymbol{\phi}} r d\phi.$$

Hence,

$$\begin{aligned} \int_{P_1}^{P_2} \mathbf{E} \cdot d\boldsymbol{\ell} &= \int_{\phi=90^\circ}^{\phi=180^\circ} \left[\hat{\mathbf{r}} r(\cos^2 \phi - \sin^2 \phi) - \hat{\boldsymbol{\phi}} 2r \sin \phi \cos \phi \right] \cdot \hat{\boldsymbol{\phi}} r d\phi \Big|_{r=3} \\ &= \int_{90^\circ}^{180^\circ} -2r^2 \sin \phi \cos \phi d\phi \Big|_{r=3} \\ &= -2r^2 \frac{\sin^2 \phi}{2} \Big|_{\phi=90^\circ}^{180^\circ} \Big|_{r=3} = 9. \end{aligned}$$
