

Problem 3.42 For the scalar function $T = \frac{1}{2}e^{-r/5} \cos \phi$, determine its directional derivative along the radial direction $\hat{\mathbf{r}}$ and then evaluate it at $P = (2, \pi/4, 3)$.

Solution:

$$\begin{aligned}
 T &= \frac{1}{2}e^{-r/5} \cos \phi, \\
 \nabla T &= \hat{\mathbf{r}} \frac{\partial T}{\partial r} + \hat{\boldsymbol{\phi}} \frac{1}{r} \frac{\partial T}{\partial \phi} + \hat{\mathbf{z}} \frac{\partial T}{\partial z} = -\hat{\mathbf{r}} \frac{e^{-r/5} \cos \phi}{10} - \hat{\boldsymbol{\phi}} \frac{e^{-r/5} \sin \phi}{2r}, \\
 \frac{dT}{dl} &= \nabla T \cdot \hat{\mathbf{r}} = -\frac{e^{-r/5} \cos \phi}{10}, \\
 \left. \frac{dT}{dl} \right|_{(2, \pi/4, 3)} &= -\frac{e^{-2/5} \cos \frac{\pi}{4}}{10} = -4.74 \times 10^{-2}.
 \end{aligned}$$
