

Problem 3.45 Vector field \mathbf{E} is characterized by the following properties: (a) \mathbf{E} points along $\hat{\mathbf{R}}$, (b) the magnitude of \mathbf{E} is a function of only the distance from the origin, (c) \mathbf{E} vanishes at the origin, and (d) $\nabla \cdot \mathbf{E} = 12$, everywhere. Find an expression for \mathbf{E} that satisfies these properties.

Solution: According to properties (a) and (b), \mathbf{E} must have the form

$$\mathbf{E} = \hat{\mathbf{R}}E_R$$

where E_R is a function of R only.

$$\begin{aligned}\nabla \cdot \mathbf{E} &= \frac{1}{R^2} \frac{\partial}{\partial R} (R^2 E_R) = 12, \\ \frac{\partial}{\partial R} (R^2 E_R) &= 12R^2, \\ \int_0^R \frac{\partial}{\partial R} (R^2 E_R) dR &= \int_0^R 12R^2 dR, \\ R^2 E_R \Big|_0^R &= \frac{12R^3}{3} \Big|_0^R, \\ R^2 E_R &= 4R^3.\end{aligned}$$

Hence,

$$E_R = 4R,$$

and

$$\mathbf{E} = \hat{\mathbf{R}}4R.$$
