

Problem 3.5 Given vectors $\mathbf{A} = \hat{\mathbf{x}} + \hat{\mathbf{y}}2 - \hat{\mathbf{z}}3$, $\mathbf{B} = \hat{\mathbf{x}}2 - \hat{\mathbf{y}}4$, and $\mathbf{C} = \hat{\mathbf{y}}2 - \hat{\mathbf{z}}4$, find

- (a) A and $\hat{\mathbf{a}}$,
- (b) the component of \mathbf{B} along \mathbf{C} ,
- (c) θ_{AC} ,
- (d) $\mathbf{A} \times \mathbf{C}$,
- (e) $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$,
- (f) $\mathbf{A} \times (\mathbf{B} \times \mathbf{C})$,
- (g) $\hat{\mathbf{x}} \times \mathbf{B}$, and
- (h) $(\mathbf{A} \times \hat{\mathbf{y}}) \cdot \hat{\mathbf{z}}$.

Solution:

- (a) From Eq. (3.4),

$$A = \sqrt{1^2 + 2^2 + (-3)^2} = \sqrt{14},$$

and, from Eq. (3.5),

$$\hat{\mathbf{a}}_A = \frac{\hat{\mathbf{x}} + \hat{\mathbf{y}}2 - \hat{\mathbf{z}}3}{\sqrt{14}}.$$

- (b) The component of \mathbf{B} along \mathbf{C} (see Section 3-1.4) is given by

$$B \cos \theta_{BC} = \frac{\mathbf{B} \cdot \mathbf{C}}{C} = \frac{-8}{\sqrt{20}} = -1.8.$$

- (c) From Eq. (3.18),

$$\theta_{AC} = \cos^{-1} \frac{\mathbf{A} \cdot \mathbf{C}}{AC} = \cos^{-1} \frac{4 + 12}{\sqrt{14}\sqrt{20}} = \cos^{-1} \frac{16}{\sqrt{280}} = 17.0^\circ.$$

- (d) From Eq. (3.27),

$$\mathbf{A} \times \mathbf{C} = \hat{\mathbf{x}}(2(-4) - (-3)2) + \hat{\mathbf{y}}((-3)0 - 1(-4)) + \hat{\mathbf{z}}(1(2) - 2(0)) = -\hat{\mathbf{x}}2 + \hat{\mathbf{y}}4 + \hat{\mathbf{z}}2.$$

- (e) From Eq. (3.27) and Eq. (3.21),

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{A} \cdot (\hat{\mathbf{x}}16 + \hat{\mathbf{y}}8 + \hat{\mathbf{z}}4) = 1(16) + 2(8) + (-3)4 = 20.$$

Eq. (3.30) could also have been used in the solution. Also, Eq. (3.29) could be used in conjunction with the result of part (d).

- (f) By repeated application of Eq. (3.27),

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{A} \times (\hat{\mathbf{x}}16 + \hat{\mathbf{y}}8 + \hat{\mathbf{z}}4) = \hat{\mathbf{x}}32 - \hat{\mathbf{y}}52 - \hat{\mathbf{z}}24.$$

Eq. (3.33) could also have been used.

- (g) From Eq. (3.27),

$$\hat{\mathbf{x}} \times \mathbf{B} = -\hat{\mathbf{z}}4.$$

(h) From Eq. (3.27) and Eq. (3.21),

$$(\mathbf{A} \times \hat{\mathbf{y}}) \cdot \hat{\mathbf{z}} = (\hat{\mathbf{x}}3 + \hat{\mathbf{z}}) \cdot \hat{\mathbf{z}} = 1.$$

Eq. (3.29) and Eq. (3.25) could also have been used in the solution.
