

**Problem 3.57** Find the Laplacian of the following scalar functions:

- (a)  $V = 4xy^2z^3$ ,
- (b)  $V = xy + yz + zx$ ,
- (c)  $V = 3/(x^2 + y^2)$ ,
- (d)  $V = 5e^{-r} \cos \phi$ ,
- (e)  $V = 10e^{-R} \sin \theta$ .

**Solution:**

(a) From Eq. (3.110),  $\nabla^2(4xy^2z^3) = 8xz^3 + 24xy^2z$ .

(b)  $\nabla^2(xy + yz + zx) = 0$ .

(c) From the inside back cover of the book,

$$\nabla^2 \left( \frac{3}{x^2 + y^2} \right) = \nabla^2(3r^{-2}) = 12r^{-4} = \frac{12}{(x^2 + y^2)^2}.$$

(d)

$$\nabla^2(5e^{-r} \cos \phi) = 5e^{-r} \cos \phi \left( 1 - \frac{1}{r} - \frac{1}{r^2} \right).$$

(e)

$$\nabla^2(10e^{-R} \sin \theta) = 10e^{-R} \left[ \sin \theta \left( 1 - \frac{2}{R} \right) + \frac{\cos^2 \theta - \sin^2 \theta}{R^2 \sin \theta} \right].$$

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