Problem 4.10 A line of charge of uniform density ρ_ℓ occupies a semicircle of radius b as shown in Fig. P4.10. Use the material presented in Example 4-4 to determine the electric field at the origin.

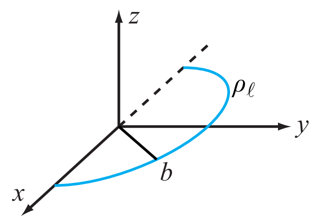

Figure P4.10: Problem 4.10.

Solution: Since we have only half of a circle, we need to integrate the expression for $d\mathbf{E}_1$ given in Example 4-4 over ϕ from 0 to π. Before we do that, however, we need to set $h = 0$ (the problem asks for $\mathbf{E}$ at the origin). Hence,

$$d\mathbf{E}_1 = \frac{\rho_\ell b}{4\pi\varepsilon_0} \left. \frac{(-\hat{\mathbf{r}}\,b + \hat{\mathbf{z}}\,h)}{(b^2+h^2)^{3/2}}\, d\phi \right|_{h=0}$$
$$= \frac{-\hat{\mathbf{r}}\,\rho_\ell}{4\pi\varepsilon_0 b}\, d\phi$$
$$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi.$$

From symmetry $\hat{\mathbf{x}}$ components cancel.

$$d\mathbf{E}_1 = \frac{-\hat{\mathbf{y}}\rho_e}{4\pi\varepsilon_0 b}\sin\phi\; d\phi,$$
$$\mathbf{E}_1 = \frac{-\hat{\mathbf{y}}\rho_e}{4\pi\varepsilon_0 b}\int_0^\pi \sin\phi\; d\phi$$
$$= \frac{\hat{\mathbf{y}}\rho_e}{4\pi\varepsilon_0 b}\cos\phi\Big|_0^\pi$$
$$= \frac{-\hat{\mathbf{y}}\rho_e}{2\pi\varepsilon_0 b}\,.$$