

Problem 4.15 Electric charge is distributed along an arc located in the x - y plane and defined by $r = 2$ cm and $0 \leq \phi \leq \pi/4$. If $\rho_\ell = 5$ ($\mu\text{C}/\text{m}$), find \mathbf{E} at $(0, 0, z)$ and then evaluate it at:

- (a) The origin.
- (b) $z = 5$ cm
- (c) $z = -5$ cm

Solution: For the arc of charge shown in Fig. P4.15, $dl = r d\phi = 0.02 d\phi$, and $\mathbf{R}' = -\hat{\mathbf{x}}0.02 \cos \phi - \hat{\mathbf{y}}0.02 \sin \phi + \hat{\mathbf{z}}z$. Use of Eq. (4.21c) gives

$$\begin{aligned} \mathbf{E} &= \frac{1}{4\pi\epsilon_0} \int_{l'} \hat{\mathbf{R}}' \frac{\rho_l dl'}{R'^2} \\ &= \frac{1}{4\pi\epsilon_0} \int_{\phi=0}^{\pi/4} \rho_l \frac{(-\hat{\mathbf{x}}0.02 \cos \phi - \hat{\mathbf{y}}0.02 \sin \phi + \hat{\mathbf{z}}z)}{((0.02)^2 + z^2)^{3/2}} 0.02 d\phi \\ &= \frac{898.8}{((0.02)^2 + z^2)^{3/2}} [-\hat{\mathbf{x}}0.014 - \hat{\mathbf{y}}0.006 + \hat{\mathbf{z}}0.78z] \quad (\text{V/m}). \end{aligned}$$

- (a) At $z = 0$, $\mathbf{E} = -\hat{\mathbf{x}}1.6 - \hat{\mathbf{y}}0.66$ (MV/m).
- (b) At $z = 5$ cm, $\mathbf{E} = -\hat{\mathbf{x}}81.4 - \hat{\mathbf{y}}33.7 + \hat{\mathbf{z}}226$ (kV/m).
- (c) At $z = -5$ cm, $\mathbf{E} = -\hat{\mathbf{x}}81.4 - \hat{\mathbf{y}}33.7 - \hat{\mathbf{z}}226$ (kV/m).

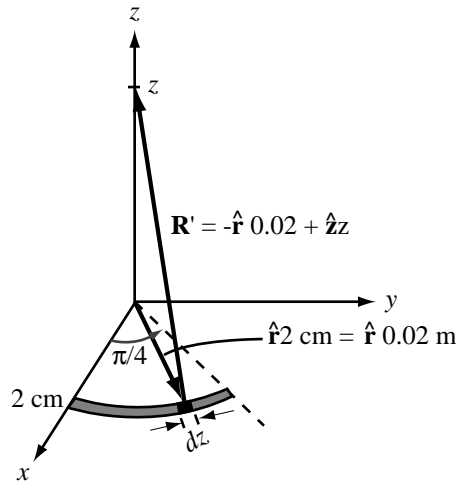


Figure P4.15: Line charge along an arc.