

Problem 4.17 Repeat Example 4-5 for the circular disk of charge of radius a , but in the present case, assume the surface charge density to vary with r as

$$\rho_s = \rho_{s0} r^2 \quad (\text{C/m}^2)$$

where ρ_{s0} is a constant.

Solution: We start with the expression for $d\mathbf{E}$ given in Example 4-5 but we replace ρ_s with $\rho_{s0} r^2$:

$$d\mathbf{E} = \hat{\mathbf{z}} \frac{h}{4\pi\epsilon_0 (r^2 + h^2)^{3/2}} (2\pi\rho_{s0} r^3 dr),$$

$$\mathbf{E} = \hat{\mathbf{z}} \frac{\rho_{s0} h}{2\epsilon_0} \int_0^a \frac{r^3 dr}{(r^2 + h^2)^{3/2}}.$$

To perform the integration, we use

$$R^2 = r^2 + h^2,$$

$$2R dR = 2r dr,$$

$$\begin{aligned} \mathbf{E} &= \hat{\mathbf{z}} \frac{\rho_{s0} h}{2\epsilon_0} \int_h^{(a^2+h^2)^{1/2}} \frac{(R^2 - h^2) dR}{R^2} \\ &= \hat{\mathbf{z}} \frac{\rho_{s0} h}{2\epsilon_0} \left[\int_h^{(a^2+h^2)^{1/2}} dR - \int_h^{(a^2+h^2)^{1/2}} \frac{h^2}{R^2} dR \right] \\ &= \hat{\mathbf{z}} \frac{\rho_{s0} h}{2\epsilon_0} \left[\sqrt{a^2 + h^2} + \frac{h^2}{\sqrt{a^2 + h^2}} - 2h \right]. \end{aligned}$$
