Problem 4.18 Multiple charges at different locations are said to be in equilibrium if the force acting on any one of them is identical in magnitude and direction to the force acting on any of the others. Suppose we have two negative charges, one located at the origin and carrying charge $-9e$, and the other located on the positive x-axis at a distance d from the first one and carrying charge $-36e$. Determine the location, polarity and magnitude of a third charge whose placement would bring the entire system into equilibrium.

Solution: If

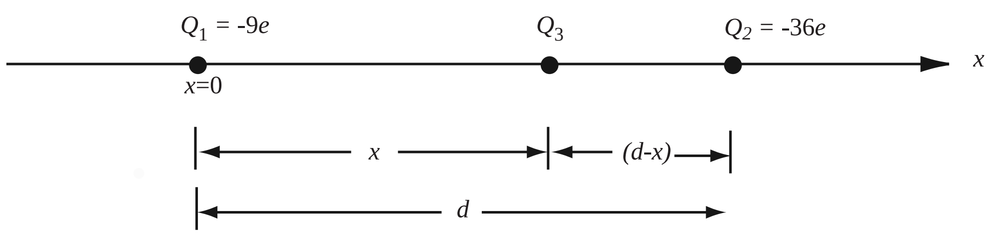

Figure P4.18: Three collinear charges.

$$\mathbf{F}_1 = \text{force on } Q_1,$$
$$\mathbf{F}_2 = \text{force on } Q_2,$$
$$\mathbf{F}_3 = \text{force on } Q_3,$$

then equilibrium means that

$$\mathbf{F}_1 = \mathbf{F}_2 = \mathbf{F}_3.$$

The two original charges are both negative, which mean they would repel each other. The third charge has to be positive and has to lie somewhere between them in order to counteract their repulsion force. The forces acting on charges Q_1, Q_2, and Q_3 are respectively

$$\mathbf{F}_1 = \frac{\hat{\mathbf{R}}_{21} Q_1 Q_2}{4\pi\varepsilon_0 R_{21}^2} + \frac{\hat{\mathbf{R}}_{31} Q_1 Q_3}{4\pi\varepsilon_0 R_{31}^2} = -\hat{\mathbf{x}} \frac{324e^2}{4\pi\varepsilon_0 d^2} + \hat{\mathbf{x}} \frac{9eQ_3}{4\pi\varepsilon_0 x^2},$$

$$\mathbf{F}_2 = \frac{\hat{\mathbf{R}}_{12} Q_1 Q_2}{4\pi\varepsilon_0 R_{12}^2} + \frac{\hat{\mathbf{R}}_{32} Q_3 Q_2}{4\pi\varepsilon_0 R_{32}^2} = \hat{\mathbf{x}} \frac{324e^2}{4\pi\varepsilon_0 d^2} - \hat{\mathbf{x}} \frac{36eQ_3}{4\pi\varepsilon_0 (d-x)^2},$$

$$\mathbf{F}_3 = \frac{\hat{\mathbf{R}}_{13} Q_1 Q_3}{4\pi\varepsilon_0 R_{13}^2} + \frac{\hat{\mathbf{R}}_{23} Q_2 Q_3}{4\pi\varepsilon_0 R_{23}^2} = -\hat{\mathbf{x}} \frac{9eQ_3}{4\pi\varepsilon_0 x^2} + \hat{\mathbf{x}} \frac{36eQ_3}{4\pi\varepsilon_0 (d-x)^2}.$$

Hence, equilibrium requires that

$$-\frac{324e}{d^2} + \frac{9Q_3}{x^2} = \frac{324e}{d^2} - \frac{36Q_3}{(d-x)^2} = -\frac{9Q_3}{x^2} + \frac{36Q_3}{(d-x)^2}.$$

Solution of the above equations yields

$$Q_3 = 4e, \qquad x = \frac{d}{3}\,.$$