

Problem 4.19 Three infinite lines of charge, all parallel to the z -axis, are located at the three corners of the kite-shaped arrangement shown in Fig. P4.19. If the two right triangles are symmetrical and of equal corresponding sides, show that the electric field is zero at the origin.

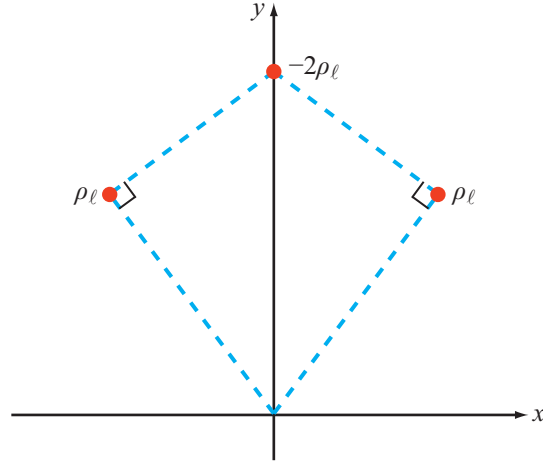


Figure P4.19: Kite-shaped arrangement of line charges for Problem 4.19.

Solution: The field due to an infinite line of charge is given by Eq. (4.33). In the present case, the total \mathbf{E} at the origin is

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3.$$

The components of \mathbf{E}_1 and \mathbf{E}_2 along $\hat{\mathbf{x}}$ cancel and their components along $-\hat{\mathbf{y}}$ add. Also, \mathbf{E}_3 is along $\hat{\mathbf{y}}$ because the line charge on the y -axis is negative. Hence,

$$\mathbf{E} = -\hat{\mathbf{y}} \frac{2\rho_l \cos \theta}{2\pi\epsilon_0 R_1} + \hat{\mathbf{y}} \frac{2\rho_l}{2\pi\epsilon_0 R_2}.$$

But $\cos \theta = R_1/R_2$. Hence,

$$\mathbf{E} = -\hat{\mathbf{y}} \frac{\rho_l}{\pi\epsilon_0 R_1} \frac{R_1}{R_2} + \hat{\mathbf{y}} \frac{\rho_l}{\pi\epsilon_0 R_2} = 0.$$
