

Problem 4.20 Three infinite lines of charge, $\rho_{l_1} = 3 \text{ (nC/m)}$, $\rho_{l_2} = -3 \text{ (nC/m)}$, and $\rho_{l_3} = 3 \text{ (nC/m)}$, are all parallel to the z -axis. If they pass through the respective points $(0, -b)$, $(0, 0)$, and $(0, b)$ in the x - y plane, find the electric field at $(a, 0, 0)$. Evaluate your result for $a = 2 \text{ cm}$ and $b = 1 \text{ cm}$.

Solution:

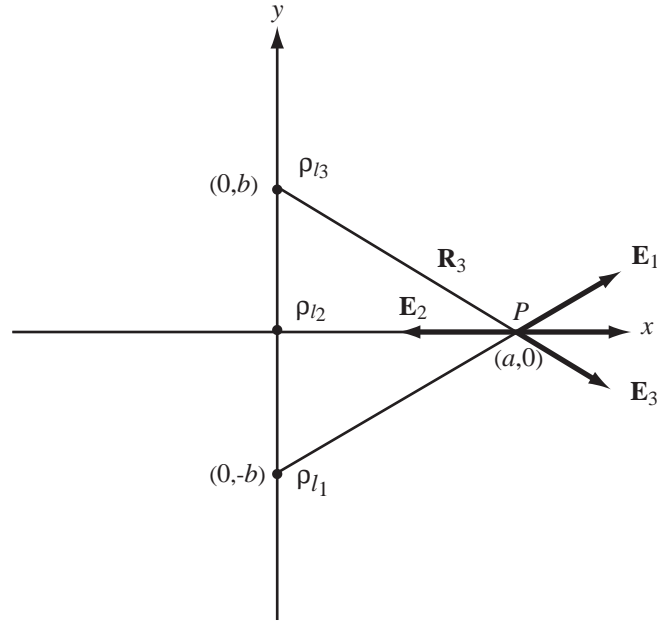


Figure P4.20: Three parallel line charges.

$$\begin{aligned}\rho_{l_1} &= 3 \text{ (nC/m)}, \\ \rho_{l_2} &= -3 \text{ (nC/m)}, \\ \rho_{l_3} &= \rho_{l_1}, \\ \mathbf{E} &= \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3.\end{aligned}$$

Components of line charges 1 and 3 along y cancel and components along x add. Hence, using Eq. (4.33),

$$\mathbf{E} = \hat{\mathbf{x}} \frac{2\rho_{l_1}}{2\pi\epsilon_0 R_1} \cos \theta + \hat{\mathbf{x}} \frac{\rho_{l_2}}{2\pi\epsilon_0 a}.$$

with $\cos \theta = \frac{a}{\sqrt{a^2 + b^2}}$ and $R_1 = \sqrt{a^2 + b^2}$,

$$\mathbf{E} = \frac{\hat{\mathbf{x}} 3}{2\pi\epsilon_0} \left[\frac{2a}{a^2 + b^2} - \frac{1}{a} \right] \times 10^{-9} \quad (\text{V/m}).$$

For $a = 2$ cm and $b = 1$ cm,

$$\mathbf{E} = \hat{\mathbf{x}} 1.62 \quad (\text{kV/m}).$$
