

Problem 4.22 Given the electric flux density

$$\mathbf{D} = \hat{\mathbf{x}}2(x+y) + \hat{\mathbf{y}}(3x-2y) \quad (\text{C/m}^2)$$

determine

- (a) ρ_v by applying Eq. (4.26).
- (b) The total charge Q enclosed in a cube 2 m on a side, located in the first octant with three of its sides coincident with the x -, y -, and z -axes and one of its corners at the origin.
- (c) The total charge Q in the cube, obtained by applying Eq. (4.29).

Solution:

- (a) By applying Eq. (4.26)

$$\rho_v = \nabla \cdot \mathbf{D} = \frac{\partial}{\partial x}(2x+2y) + \frac{\partial}{\partial y}(3x-2y) = 0.$$

- (b) Integrate the charge density over the volume as in Eq. (4.27):

$$Q = \int_V \nabla \cdot \mathbf{D} \, dV = \int_{x=0}^2 \int_{y=0}^2 \int_{z=0}^2 0 \, dx \, dy \, dz = 0.$$

- (c) Apply Gauss' law to calculate the total charge from Eq. (4.29)

$$\begin{aligned} Q &= \oint \mathbf{D} \cdot d\mathbf{s} = F_{\text{front}} + F_{\text{back}} + F_{\text{right}} + F_{\text{left}} + F_{\text{top}} + F_{\text{bottom}}, \\ F_{\text{front}} &= \int_{y=0}^2 \int_{z=0}^2 (\hat{\mathbf{x}}2(x+y) + \hat{\mathbf{y}}(3x-2y)) \Big|_{x=2} \cdot (\hat{\mathbf{x}} \, dz \, dy) \\ &= \int_{y=0}^2 \int_{z=0}^2 2(x+y) \Big|_{x=2} \, dz \, dy = \left(2z \left(2y + \frac{1}{2}y^2 \right) \Big|_{z=0}^2 \right) \Big|_{y=0}^2 = 24, \\ F_{\text{back}} &= \int_{y=0}^2 \int_{z=0}^2 (\hat{\mathbf{x}}2(x+y) + \hat{\mathbf{y}}(3x-2y)) \Big|_{x=0} \cdot (-\hat{\mathbf{x}} \, dz \, dy) \\ &= - \int_{y=0}^2 \int_{z=0}^2 2(x+y) \Big|_{x=0} \, dz \, dy = - \left(zy^2 \Big|_{z=0}^2 \right) \Big|_{y=0}^2 = -8, \\ F_{\text{right}} &= \int_{x=0}^2 \int_{z=0}^2 (\hat{\mathbf{x}}2(x+y) + \hat{\mathbf{y}}(3x-2y)) \Big|_{y=2} \cdot (\hat{\mathbf{y}} \, dz \, dx) \\ &= \int_{x=0}^2 \int_{z=0}^2 (3x-2y) \Big|_{y=2} \, dz \, dx = \left(z \left(\frac{3}{2}x^2 - 4x \right) \Big|_{z=0}^2 \right) \Big|_{x=0}^2 = -4, \end{aligned}$$

$$\begin{aligned}
F_{\text{left}} &= \int_{x=0}^2 \int_{z=0}^2 (\hat{\mathbf{x}}2(x+y) + \hat{\mathbf{y}}(3x-2y)) \Big|_{y=0} \cdot (-\hat{\mathbf{y}} \, dz \, dx) \\
&= - \int_{x=0}^2 \int_{z=0}^2 (3x-2y) \Big|_{y=0} dz \, dx = - \left(z \left(\frac{3}{2}x^2 \right) \Big|_{z=0}^2 \right) \Big|_{x=0}^2 = -12,
\end{aligned}$$

$$\begin{aligned}
F_{\text{top}} &= \int_{x=0}^2 \int_{z=0}^2 (\hat{\mathbf{x}}2(x+y) + \hat{\mathbf{y}}(3x-2y)) \Big|_{z=2} \cdot (\hat{\mathbf{z}} \, dy \, dx) \\
&= \int_{x=0}^2 \int_{z=0}^2 0 \Big|_{z=2} dy \, dx = 0,
\end{aligned}$$

$$\begin{aligned}
F_{\text{bottom}} &= \int_{x=0}^2 \int_{z=0}^2 (\hat{\mathbf{x}}2(x+y) + \hat{\mathbf{y}}(3x-2y)) \Big|_{z=0} \cdot (\hat{\mathbf{z}} \, dy \, dx) \\
&= \int_{x=0}^2 \int_{z=0}^2 0 \Big|_{z=0} dy \, dx = 0.
\end{aligned}$$

Thus $Q = \oint \mathbf{D} \cdot d\mathbf{s} = 24 - 8 - 4 - 12 + 0 + 0 = 0$.
