

Problem 4.27 An infinitely long cylindrical shell extending between $r = 1$ m and $r = 3$ m contains a uniform charge density ρ_{v0} . Apply Gauss's law to find \mathbf{D} in all regions.

Solution: For $r < 1$ m, $\mathbf{D} = 0$.

For $1 \leq r \leq 3$ m,

$$\oint_S \hat{\mathbf{r}} D_r \cdot d\mathbf{s} = Q,$$
$$D_r \cdot 2\pi r L = \rho_{v0} \cdot \pi L (r^2 - 1^2),$$
$$\mathbf{D} = \hat{\mathbf{r}} D_r = \hat{\mathbf{r}} \frac{\rho_{v0} \pi L (r^2 - 1)}{2\pi r L} = \hat{\mathbf{r}} \frac{\rho_{v0} (r^2 - 1)}{2r}, \quad 1 \leq r \leq 3 \text{ m}.$$

For $r \geq 3$ m,

$$D_r \cdot 2\pi r L = \rho_{v0} \pi L (3^2 - 1^2) = 8\rho_{v0} \pi L,$$
$$\mathbf{D} = \hat{\mathbf{r}} D_r = \hat{\mathbf{r}} \frac{4\rho_{v0}}{r}, \quad r \geq 3 \text{ m}.$$

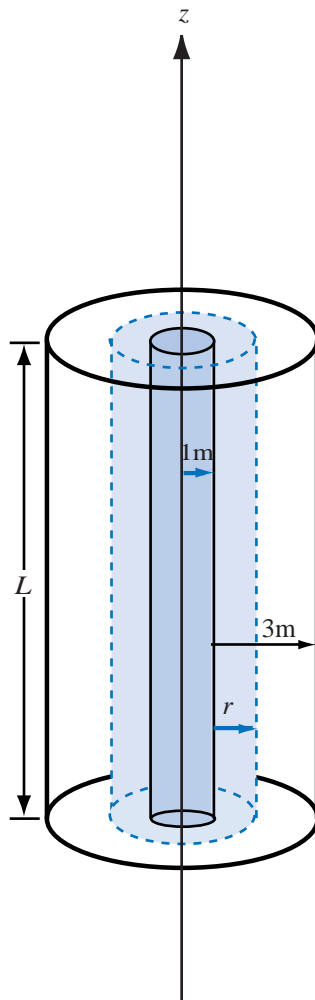


Figure P4.27: Cylindrical shell.
