**Problem 4.29** A spherical shell with outer radius $b$ surrounds a charge-free cavity of radius $a < b$ (Fig. P4.29). If the shell contains a charge density given by

$$\rho_v = -\frac{\rho_{v0}}{R^2}, \qquad a \le R \le b,$$

where $\rho_{v0}$ is a positive constant, determine $\mathbf{D}$ in all regions.

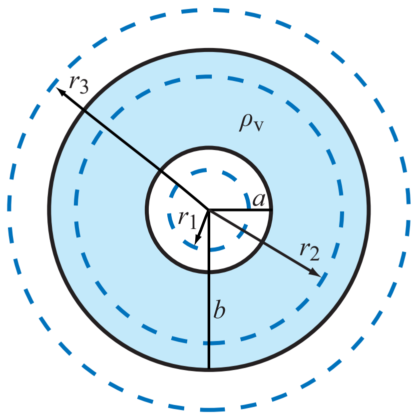


**Figure P4.29:** Problem 4.29.

**Solution:** Symmetry dictates that $\mathbf{D}$ is radially oriented. Thus,

$$\mathbf{D} = \hat{\mathbf{R}} D_R.$$

At any $R$, Gauss's law gives

$$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q$$

$$\int_S \hat{\mathbf{R}} D_R \cdot \hat{\mathbf{R}}\, ds = Q$$

$$4\pi R^2 D_R = Q$$

$$D_R = \frac{Q}{4\pi R^2}.$$

**(a)** For $R < a$, no charge is contained in the cavity. Hence, $Q = 0$, and

$$D_R = 0, \qquad R \le a.$$

**(b)** For $a \le R \le b$,

$$Q = \int_{R=a}^{R} \rho_v \, dV = \int_{R=a}^{R} -\frac{\rho_{v0}}{R^2} \cdot 4\pi R^2 \, dR$$
$$= -4\pi \rho_{v0}(R - a).$$

Hence,

$$D_R = -\frac{\rho_{\mathrm{v}0}(R-a)}{R^2}\,, \qquad a \le R \le b.$$

**(c)** For $R \ge b$,

$$Q = \int_{R=a}^{b} \rho_{\mathrm{v}}\, dV = -4\pi\rho_{\mathrm{v}0}(b-a)$$

$$D_R = -\frac{\rho_{\mathrm{v}0}(b-a)}{R^2}\,, \qquad R \ge b.$$