

Problem 4.30 A square in the x - y plane in free space has a point charge of $+Q$ at corner $(a/2, a/2)$, the same at corner $(a/2, -a/2)$, and a point charge of $-Q$ at each of the other two corners.

(a) Find the electric potential at any point P along the x -axis.

(b) Evaluate V at $x = a/2$.

Solution: $R_1 = R_2$ and $R_3 = R_4$.

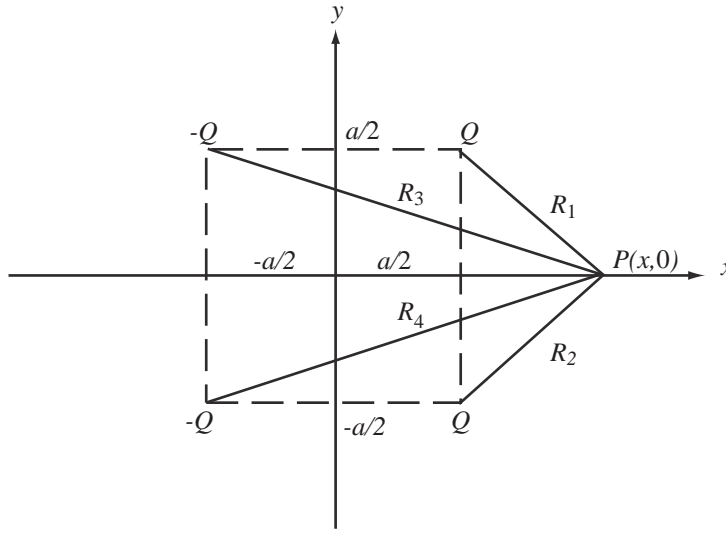


Figure P4.30: Potential due to four point charges.

$$V = \frac{Q}{4\pi\epsilon_0 R_1} + \frac{Q}{4\pi\epsilon_0 R_2} + \frac{-Q}{4\pi\epsilon_0 R_3} + \frac{-Q}{4\pi\epsilon_0 R_4} = \frac{Q}{2\pi\epsilon_0} \left(\frac{1}{R_1} - \frac{1}{R_3} \right)$$

with

$$R_1 = \sqrt{\left(x - \frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2},$$

$$R_3 = \sqrt{\left(x + \frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2}.$$

At $x = a/2$,

$$R_1 = \frac{a}{2},$$

$$R_3 = \frac{a\sqrt{5}}{2},$$
$$V = \frac{Q}{2\pi\epsilon_0} \left(\frac{2}{a} - \frac{2}{\sqrt{5}a} \right) = \frac{0.55Q}{\pi\epsilon_0 a}.$$
